Double Star Astronomy

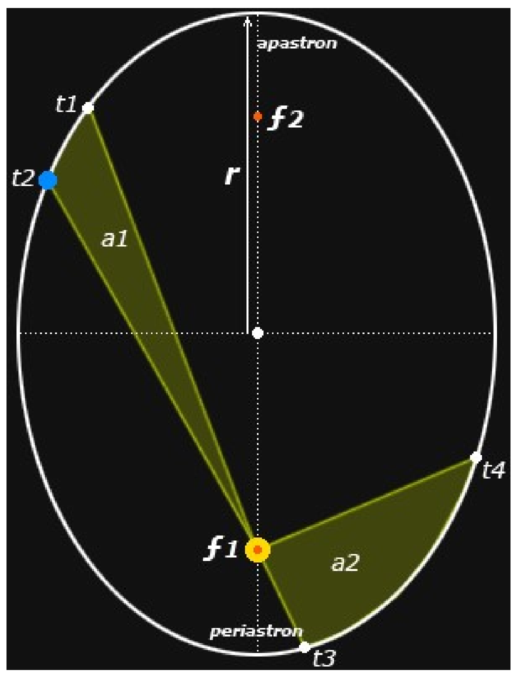
Part 4: Orbital & Dynamic Elements

Historically, the orbital motions of double stars provided the first evidence that Newton's description of gravitational attraction and the laws of motion (in the *Principia mathematica,* 1687) applied not just to the planets and periodic comets of the solar system but equally to the celestial motions of the stars around each other and around the Galaxy. The 19th century analysis of the orbits of a small number of nearby, short period binary stars, many of them eclipsing variable and spectroscopic stars, revealed the variety of stellar masses and dimensions, which laid the foundation for theories of stellar structure and evolution.

I first present the classical view of Kepler and Newton, then how to "build" a binary star from mass, orbital radius and orbital energy. The orbital elements are defined and illustrated in the graphical construction of the relative orbit and the method used to recreate the inclination.

Gravitational Dynamics

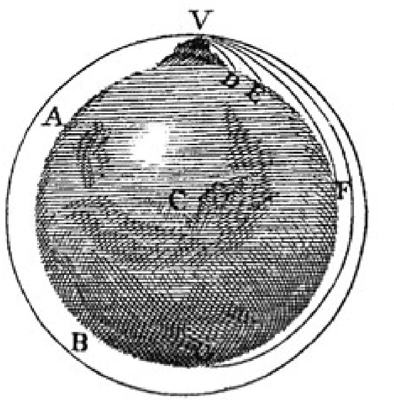
Kepler's Laws. The effects of gravity within the solar system were first presented in the *Epitome of Copernican Astronomy, Books IV & V* (1621) by Johannes Kepler. By analyzing measurements of the motion of Mars made by Tycho Brahe, Kepler deduced his three principles of planetary motion (diagram, below):



> First Law. The orbit of every planet is an ellipse with the Sun at one of the two focal points of the ellipse.
>
> The Sun or more massive star is located at the focus ƒ1, and the orbit describes the motion of a planet or the less massive star in a binary.
>
> Second Law. A line from the star at ƒ1 to another star or planet sweeps over equal areas in equal intervals of time.
>
> Therefore the ratio between two areas swept out by a planet is equal to the ratio between the two time intervals: a1/a2 = (t1-t2)/(t3-t4). This describes orbital velocity as greatest at *periastron* or smallest orbital separation between the two bodies, and slowest at *apastron* or point of largest orbital separation.
>
> Third Law. The square of the orbital period of a planet is proportional to the cube of the semimajor axis of its orbit.
>
> The semimajor axis is the distance **r** measured from the center of the ellipse to the point of periastron or apastron. If the ellipse is a circle, **r** is the radius of the circle.

These are often imprecisely called Kepler's "Laws," although they are not physical laws in the scientific sense but empirical principles or generalizations. However they are the phenomena that scientific laws must explain.

Newton's Mechanics. The geometric formulation of the laws of motion described by Galileo was accomplished by Isaac Newton's *Philosophiae Naturalis Principia Mathematica* (1687) — the *mathematical principles of natural philosophy,* as science was then called.



Newton's "thought experiment" was to imagine a powerful cannon at the peak of a very high mountain (at V, diagram left). According to Newton's first law of motion, a cannonball fired from the perfectly level cannon would tend to travel forever in a straight line at a fixed velocity and kinetic energy. But the continuous downward pull of Earth's gravity would bend the path into a parabolic trajectory until the cannonball hit the Earth at D.

If the powder charge in the cannon were increased, the initial velocity of the cannonball would be greater, its kinetic energy would be greater, and it would travel farther, to E or even to F. Eventually, if enough powder were used to impart a sufficiently high initial velocity, the cannonball would circle the Earth and return to V in a *closed orbit.*

This illustrates that planetary orbits are possible because the orbital velocity balances the gravitational acceleration, and also suggests that *circular orbits contain the minimum orbital velocity* or lowest energy for a given orbital radius. Higher energy orbits would be increasingly elliptical, up to the point where the orbital energy was sufficient to produce an *escape velocity* and the observed section of the trajectory or "orbit" would be in the form of a parabola or hyperbola.

Newton showed by a geometrical proof (not by the calculus that he invented for numerical analysis) that an elliptical orbit must be produced by an *inverse square* mutual attraction between two orbiting bodies:

$$\mathbf{F}_{d2} = \mathbf{F}_{d1}\cdot(\mathbf{d1}/\mathbf{d2})^2$$

As the distance between two bodies is changed, the gravitational attraction between them is changed by the square of the ratio of the distances. The corresponding kinetic energy necessary to sustain the orbit is changed in the same proportion.

The Dynamical Equations. Newton's key insight was that gravity was a force continuously exerted on masses, and was therefore a form of *acceleration.* This linked it directly to his definition of force as exerted in the simplest case of a circular orbit that will have a constant radius and orbital velocity:

$$\mathbf{F} = \mathbf{ma} = \mathbf{mv}^2/\mathbf{r}$$

where the acceleration due to gravity (**a**) is measured as the constant orbital velocity squared ($\mathbf{v}^2$, in meters per second) divided by the orbital radius (**r**, in meters). Because the force is the gravitational constant $\mathbf{G} = 6.674 \times 10^{-11}$ kg^{-1} / $\mathbf{m}^3$ / sec^{-2}, the measured radius and velocity create a ratio with the gravitational constant that reveals the system mass (**m**, in kilograms):

$$\mathbf{m} = \mathbf{rv}^2/\mathbf{G}$$

For rapidly orbiting spectroscopic binaries, the orbital velocity can be measured directly from the maximum observed Doppler shift in the spectral lines of the individual stars, with a correction applied for the tilt of the orbit to our line of sight.

For orbital velocities that are too slow or tilted too far to the line of sight to provide a measurable velocity, the period can be estimated from an *orbital solution* based on the changing position of the components measured across years or decades and a parallax estimate of the system distance, which yields the orbital radius. Then:

$$\mathbf{v}^2/\mathbf{r} = 2\pi\mathbf{r}/\mathbf{P}$$

so that the necessary force is now defined as:

$$\mathbf{G} = 4\pi^2\mathbf{mr}/\mathbf{P}^2$$

Finally, Kepler's Third Law, $\mathbf{P} \propto \mathbf{r}^{3/2}$, generalizes to elliptical orbits, and gives

$$\mathbf{G} = 4\pi^2\mathbf{r}^3/(\mathbf{M}_1+\mathbf{M}_2)\mathbf{P}^2$$

where the masses of the two orbiting bodies are $\mathbf{M}_1$ and $\mathbf{M}_2$.

The Solar Standard Formulas. Because the Earth is only about 0.0001% (one millionth) the mass of the Sun, the mass of the combined system is effectively the mass of the Sun, and the Earth's period at the Earth's average orbital radius is effectively a measure of the solar mass. This means the dimensions of the solar system can provide units of measurement that are already standardized on the gravitational constant, so it can be dropped from the equations.

If solar standard units are used — the astronomical unit (AU) for the semimajor axis **r**, solar mass $\mathbf{M}_\odot$ for the combined mass of both components, and years for the orbital period **P** — then the three possible versions of Kepler's Third Law simplify into the elegant:

$$\mathbf{P}_{years} = [\ \mathbf{r}_{AU}{}^3/(\mathbf{M}_1+\mathbf{M}_2)_\odot\ ]^{1/2}$$

$$\mathbf{r}_{AU} = [\ \mathbf{P}^2{}_{years}\cdot(\mathbf{M}_1+\mathbf{M}_2)_\odot\ ]^{1/3}$$

$$(\mathbf{M}_1+\mathbf{M}_2)_\odot = \mathbf{r}^3{}_{AU}/\mathbf{P}^2{}_{years}$$

In the case where the observed orbit is too slow to yield an orbital solution, the relative mass of the two components of the system can be estimated from their apparent magnitudes. Assuming that both stars are on the main sequence (and therefore have a luminosity that corresponds to the mass), the system *mass ratio* (**q**) is estimated as:

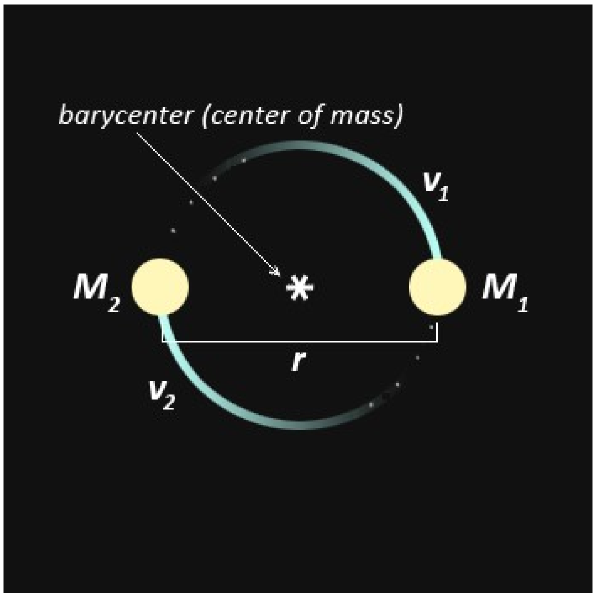
$$\mathbf{q} = 10^{-(M2-M1)/10}$$

where **M**2 and **M**1 are the absolute magnitudes of the fainter and brighter star in the pair (so that the exponent is always either zero or a negative fraction). Thus two stars of equal magnitude and spectral type have equal masses; a pair that differs by one magnitude has an estimated mass ratio of $\mathbf{q} = 10^{-1/10}$ or roughly 0.8; a two magnitude difference yields **q** = 0.6, and a three magnitude difference **q** = 0.5.

The fact that the orbital dynamics are determined by the mass of the components, and a parallax estimate of distance yields the absolute luminosity of the components, allowed the stellar **mass/luminosity relation** to be determined through the painstaking, century long measurement of a small number of eclipsing variable stars, spectroscopic binaries and closely orbiting visual double stars within a few hundred parsecs of the Earth.

Building a Binary Orbit

The most effective way to understand the binary orbit is to build one — from the simplest possible to the more complex.

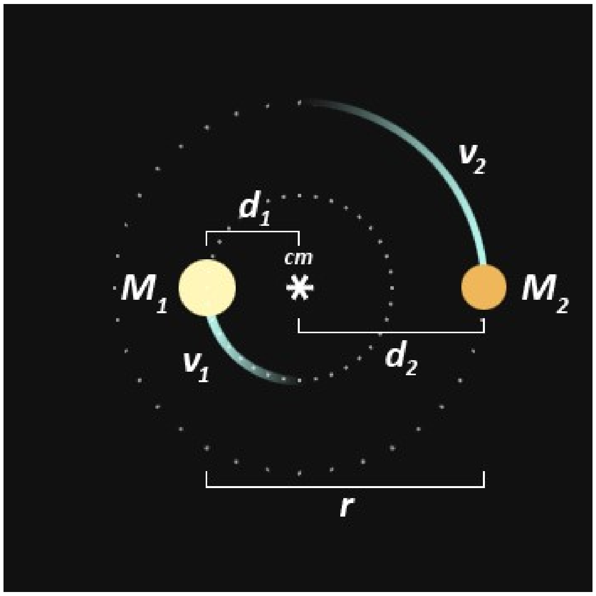


1 The simplest possible binary system consists of two identical stars in a perfectly circular orbit. Circular orbits are mostly found in close orbiting binaries with periods of around two weeks or less. A classic example is the eclipsing variable star beta Lyrae with a period of 13 days.

The total system mass is $\mathbf{M}_1+\mathbf{M}_2$. To calculate the orbital period using Kepler's third law, we use the distance between the two stars as the orbital radius (*r*): this distance, in combination with the system mass, determines the amount of gravitational force acting on the system.

However, the two stars do not orbit one around the other. Instead, both orbit around their common center of mass or *barycenter* at the center of their shared orbit and always on a line between them. This means they have the same orbital period.

Because the orbital radius is constant the gravitational force is constant, so the stars orbit at a constant orbital velocity: $\mathbf{v}_1 = \mathbf{v}_2$. A circular orbit contains the lowest orbital kinetic energy for orbital radius: *all the orbital energy* is contained in the angular momentum.



2 This simplest of all possible binaries can be complicated in two ways. First, in the vast majority of double stars, the two components are of unequal mass.

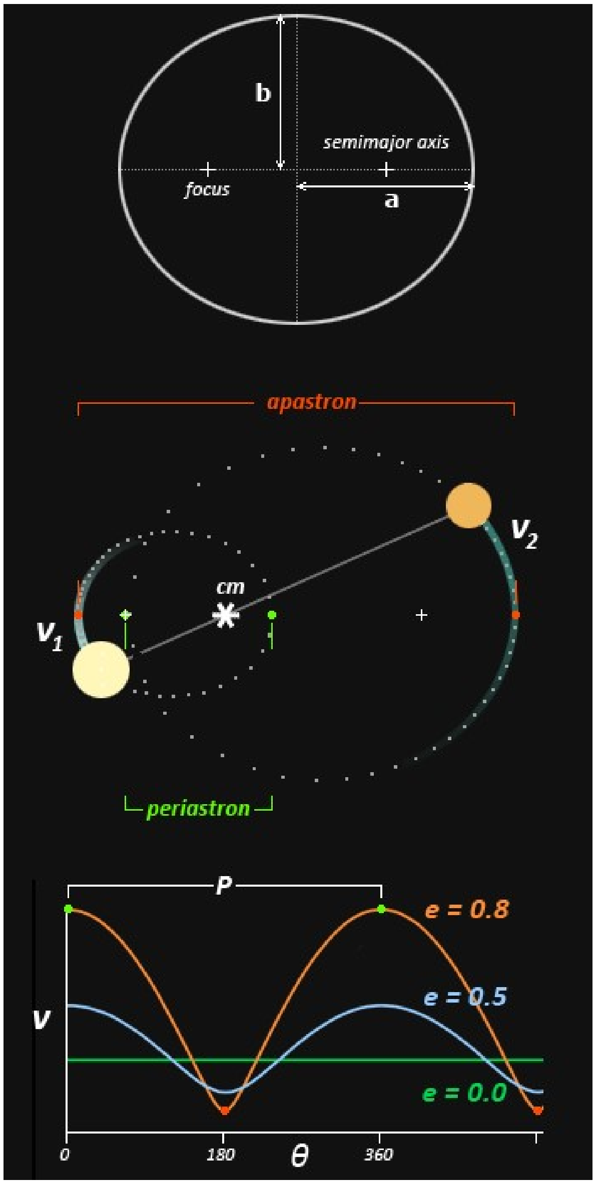
The two stars still follow circular orbits, but the relative distance of the stars from their center of mass is proportional to the *mass ratio*, $\mathbf{M}_2/\mathbf{M}_1$, of the components:

$d^1/d^2 = M^2/M^1$

In the same way that a heavier weight must be placed closer to the fulcrum of a balance beam, the heavier star must be closer to the barycenter. As a result, the more massive star orbits entirely inside the orbit of the less massive star.

The orbital radius as used in Kepler's third law is still the distance between the stars; the two stars are still connected by a line through the barycenter; they orbit in the same plane; they have the same orbital period. Because the more massive star has a smaller orbit it has a lower orbital velocity, again proportional to the mass ratio:

$\mathbf{v}_1/\mathbf{M}_2 = \mathbf{v}_2/\mathbf{M}_1$.



3 The second complication, also found in the vast majority of known double stars, is that the total orbital energy is larger than the angular momentum of a circular orbit. This excess energy causes the orbital radius to oscillate in synchrony with the orbital period, which sends the two stars into opposing elliptical orbits, defined by the orbital *eccentricity* (e):

$$e = (1 - b^2/a^2)^{1/2}$$

where a is the *semimajor axis* of the ellipse, half the longest dimension (diagram, top left).

The diagram (middle left) shows a system of eccentricity 0.5, which is about average for all binary stars. Their common center of mass is located at one focus of each orbital ellipse.

Six features define the relationship between the barycenter and the separate orbits of the binary components: (1) the two stars are always connected by a line through this fulcrum point, (2) both component orbits and the barycenter lie in a single plane, (3) both components orbit in the same direction. (4) both have the same orbital period, (5) the relative distances of the components from the barycenter and the relative size of their average orbital radius (r) are always equal to the system *mass ratio,* and (6) both orbits have the same eccentricity. The more massive star orbits more slowly in a proportionately smaller orbit. The actual distance (d) of each component from the barycenter, for any radial angle d_θ measured in a cartesian plane with the origin at the barycenter of the system, is determined by the *shape equation:*

$$d = a \cdot (1 - e^2)/[1 + (e \cdot \text{cosine}(\theta))]$$

and

$$X = d \cdot \text{cosine}(\theta)$$
$$Y = d \cdot \text{sine}(\theta)$$

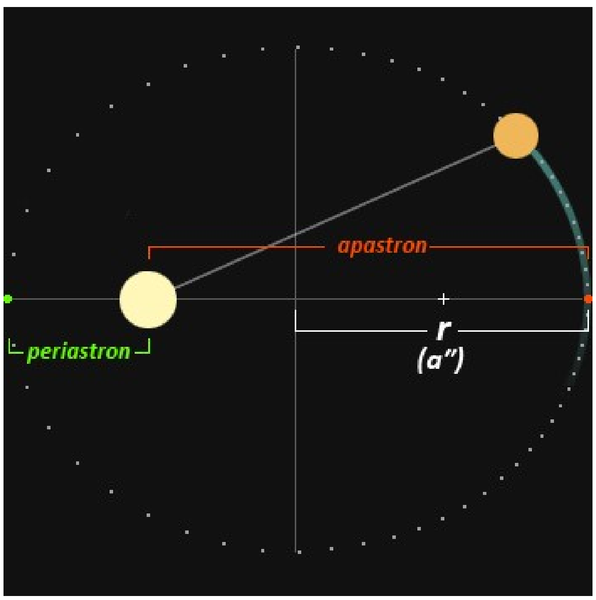
The elliptical orbits produce a continuous change in the distance between the two stars — the synchronous orbital oscillation — from a point of maximum separation or *apastron* to a point of minimum separation or *periastron.* Time related orbital attributes are usually measured from the *time of periastron passage,* — at that point the stars are closest and also moving most rapidly so the point can be observed most accurately.

Because the distance between the stars changes, their orbital velocities must change to match the changing force of gravitational attraction (diagram, left bottom). This varies with the distance (d) of each component from the barycenter:

$$v^2 = GM(2/d - 1/a) \approx 1/d$$

The plot of velocity on orbital angle (θ) shows that a circular orbit has constant velocity, and an eccentric orbital velocity follows an approximate sine wave, but with a narrowed peak at the lowest velocity (apastron) and a broadening of the curve at high velocity (periastron). In fact, it takes each component a longer time to pass through the apastron rather than periastron half of the orbital ellipse, as shown by the equal time spacing of the orbital dots in the diagram. As the eccentricity of the absolute orbit increases, this narrowing and broadening of the velocity curve becomes more pronounced. Kinetic orbital energy is transformed into potential energy en route to apastron, and the orbit is bound so long as the minimum orbital velocity is less than the escape velocity. All the dynamics are driven by oscillations between kinetic and potential energy: at all times the angular momentum of the components is conserved.

Although elliptical orbits are by far the most common, all the orbits in 1 to 3 represent the **absolute orbit** of a binary star, the dynamical pattern of their motions as observed from a frame of reference comoving with the barycenter of the system.



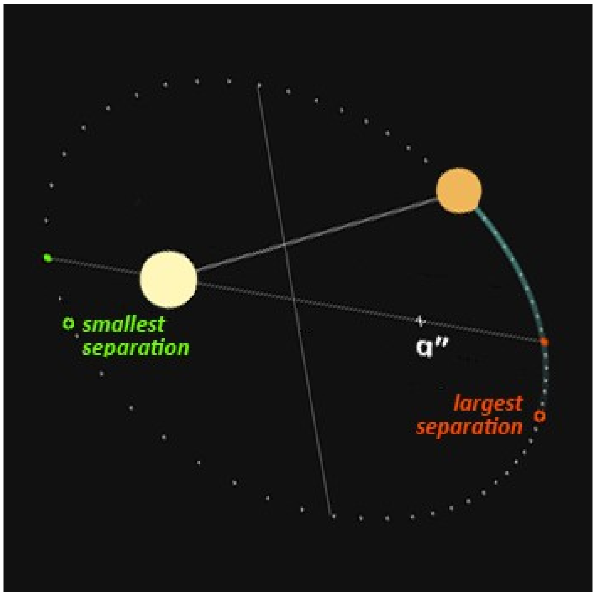
4 Unfortunately the barycenter of a binary system is invisible, so we cannot use it as a reference point to measure the separate orbital motions. Instead, we simply assume that our frame of reference is anchored on the primary (more massive) star, and measure the movement of the secondary star in relation to it.

This produces a mathematically much more convenient **relative orbit** (sometimes misleadingly called the *true orbit*) which has the same eccentricity and orbital period as the absolute orbit but always has a larger dimension: its major axis or longest dimension is the sum of the periastron and apastron distances, whereas the longest dimension of the absolute orbit is the apastron distance alone.

The *average orbital radius* (r) is now half the longest dimension or *semimajor axis* (a) of the ellipse, and this is the radius distance used in Kepler's third law. However, because we often do not know the precise distance to a double star, the semimajor axis (a) is given in arcseconds — as it would be measured on the sky if the ellipse of the relative orbit were visible. If the distance (D) in parsecs is known, then we can convert a (in arcseconds) to r (in astronomical units):

$$r = aD$$

To define the relative orbit, visual double stars are measured as the position angle and distance in arcseconds of the smaller star in relation to the larger. But the relative orbit is not simply a measurement convenience: the entire apparatus of orbital calculations, like Kepler's Laws, assumes this simplified orbital geometry.



5 A final complication does not arise in the binary orbit itself but in our point of view when we measure it.

Nearly always, the plane of the absolute and relative orbits, the semimajor axis of the relative orbit, and the angular separation between the components, are tilted in relation to our direction of view. This can radically alter both the apparent eccentricity and measured dimensions of the orbit. The points at which the two components are either closest or farthest apart are no longer the periastron or apastron, the apparent separation is typically less than the actual separation, and the eccentricity of the orbit is different (diagram, left).

Complex mathematics are necessary to correct for the foreshortened dimensions and retrieve the relative orbit in its true proportions, and they depend critically on our estimate of the inclination (i) and line of nodes (ω) of the orbit in relation to the relative orbit. In the diagram (right), the orbit is inclined 45° to our line of sight (i = 45° or 135°), on a line of nodes that is (in the relative orbit) 45° from the minor axis of the ellipse.

To summarize, binary stars can be represented in one of three ways:

(1) The *absolute orbit* or joint physical motion of the two stars in a reference frame comoving with the center of mass of the binary system, from a viewpoint perpendicular to the orbital plane of the components;
(2) The *apparent orbit* of the two stars in a reference plane tangent to the celestial sphere at the primary star, and measured assuming the primary star is fixed and the secondary orbits around it;
(3) The *relative orbit* (sometimes called the *true orbit*), which is a transformation of the apparent orbit as it would appear if the binary orbital plane were tangent to the celestial sphere.

As the center of mass, the barycenter traces the galactic orbital trajectory of the binary system which, if it were visible, would appear as a straight line proper motion across the celestial sphere. In closely orbiting, short period binaries, the two components of the system appear to oscillate or "wiggle" back and forth around this straight line path. If the second component is too faint to be optically visible, the direction and pace in the proper motion of the primary star will appear to change periodically, and these perturbations allow the presence and mass of the secondary to be estimated. Both Sirius and Procyon were first identified as binary stars in this way.

What About Triple Stars? Are binary orbits the most complex possible? What about triple, quadruple, quintuple stars?

The answer is that, in nearly all cases where stable multiple systems have been identified, the orbits are **dynamically segregated** binary orbits. If it is a triple star, then the third (single) component orbits the binary at a much greater orbital radius than the binary, forming a "binary" of a binary and single component. If it is a quadruple star comprising two binaries, then the binaries orbit their common barycenter at much greater distances than the orbits of either binary, in effect forming a "binary" of two binary components ... and so on.

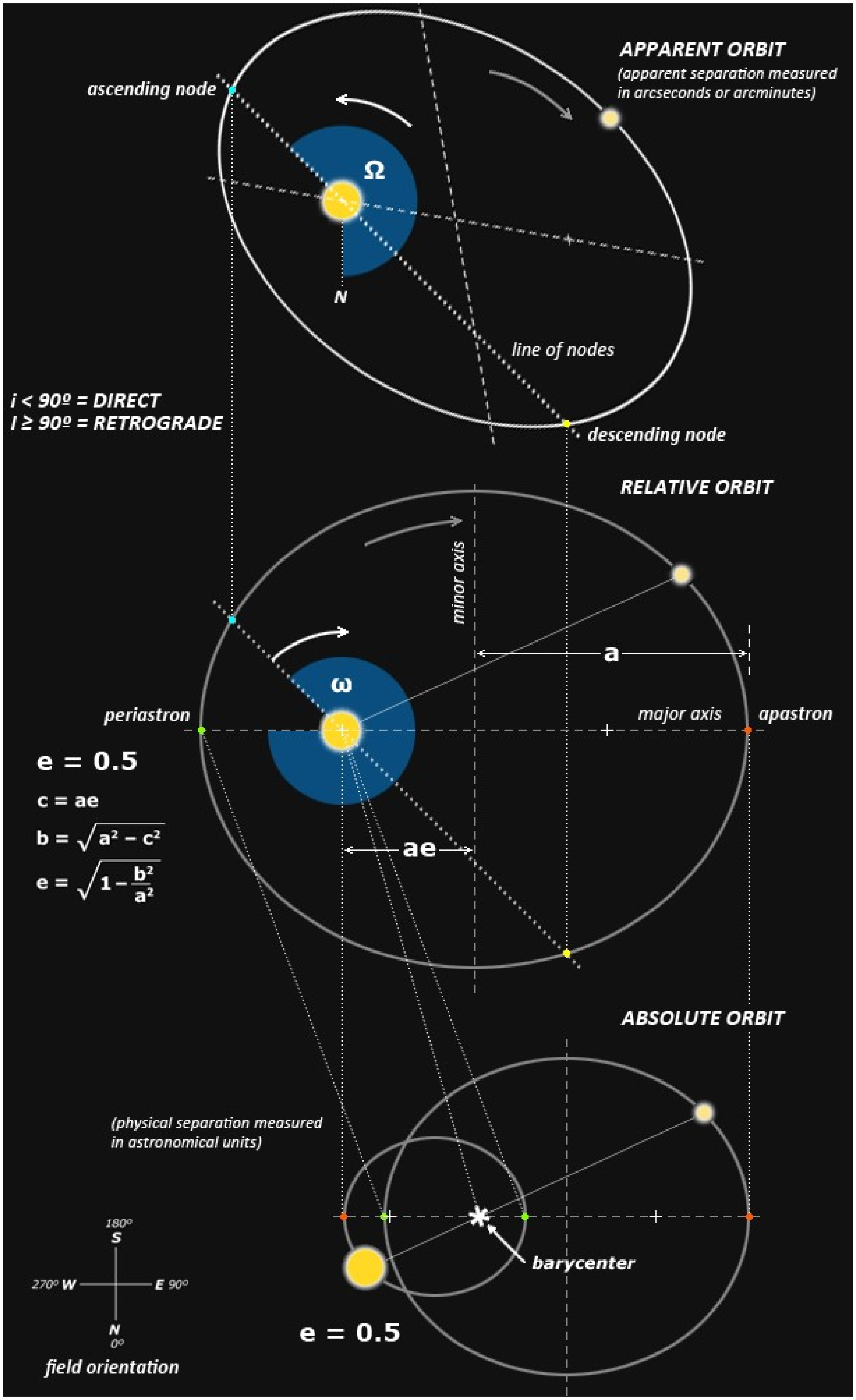
The basic principle is that orbits are spaced dynamically so that the inner orbits are not perturbed by the motions of the outer components. How far apart is far enough? Observations of multiple stars in the solar neighborhood suggest the separations are 100 to 1000 times the separation inside the binary unit, and computer simulations suggest that these systems can be both stable and bound with an outer orbital radius of 100,000 AU or more.

Current theories of star formation suggest that multiple stars form as a result of turbulent fragmentation inside the same collapsing cloud core, and computer simulations show that triple stars born in such close proximity will dynamically "unfold" into a binary plus single or 2+1 system by transferring angular momentum from the binary pair (making their orbit smaller) to the singleton (making its orbit larger, more energetic and typically more elliptical).

There are a few arcane orbital configurations of three stars that can coexist in close orbit with each other, but it is difficult to see how these would form naturally. Instead, multiple stars that cannot reach a stable segregation of orbital energies are most likely to break apart, always by keeping the binary elements intact.

Double Star Orbital Elements

The orbits of binary systems can be analyzed if sufficiently accurate positional (or *visual*) measurements of angular separation and position angle are available across a substantial part of the orbital path. In general the most accurately described orbits have an inclination that is not close to 0° or 180° and have been measured over more than half the complete orbital period.



The diagram (above) summarizes the relationships between the absolute, relative (or "true") and apparent orbits, using the calculated orbit of iota Leonis as an example. The key constants, indicated by the dotted lines connecting the different orbits, are: (1) the angular separation or apparent distance between the components at every point in the orbital cycle (including apastron and periastron) is identical between the absolute and relative orbits; and (2) the angular width of the line of nodes (between the ascending and descending nodes) is identical between the relative and apparent orbits.

Distances between the components in the apparent orbit are described in units of angular width (such as arcseconds or arcminutes), as these are the units of the visual measurements; arcseconds are also used to describe the semimajor axis of the calculated relative orbit. Distances between the components in the absolute orbit are described in terms of astronomical units (or kilometers), and separation in astronomical units can also be applied to the relative orbit, simply by multiplying the arcsecond length of the semimajor axis (a) by the distance of the system in parsecs.

Note that the angular dimension of the secondary orbit major axis is always smaller in the absolute than in the relative orbit, although the eccentricity of the orbits is the same; and that the eccentricity of the orbits is generally not the same between the relative and apparent orbits. In addition, the points where a binary star apparent orbit presents the smallest and largest angular separation (green dots in diagram) are typically not the apastron and periastron of the relative orbit, and the two points typically do not lie on a line through the primary star. This means the ephemeride date of periastron passage will not indicate the time of closest visual separation.

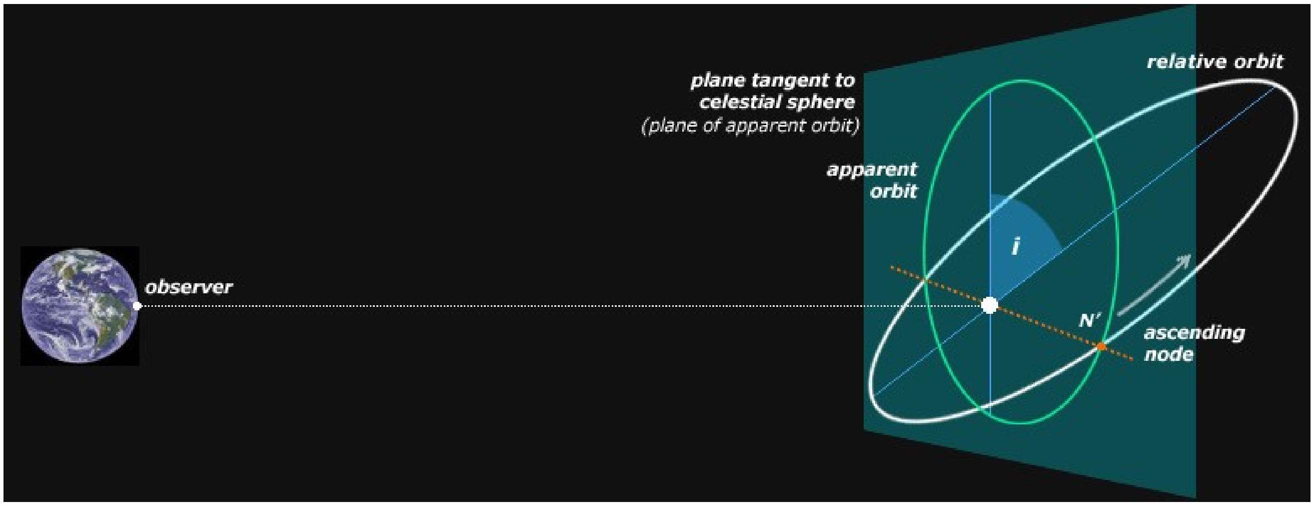
The table (below) indicates the principal orbital elements in the apparent orbit and relative orbit (sometimes called the *true orbit*).

element	symbol	apparent orbit	relative orbit
Dynamical Elements			
period	P	the time for the system to complete one sidereal revolution	
mean motion	n	= 360°/P	
periastron	.	*the projection of this point*	the point in the relative orbit where the distance between the two stars is smallest and orbital speed is greatest
time of periastron passage	T	date and/or time of the (usually most recent) periastron passage of the two stars	
eccentricity	e	.	the deviation of the relative orbit from a circle, calculated as $\mathbf{e} = \sqrt{1-(\mathbf{b}/\mathbf{a})^2}$
semimajor axis	a	*the projection of this line*	the distance (usually in arcseconds) in the relative orbit from the center C to the orbit at periastron or apastron; equivalent to the projected average orbital radius.
Campbell Elements			
orbital inclination	i	*the direction of secondary rotation:* ***i** < 90° = direct (counterclockwise)* ***i** ≥ 90° = retrograde (clockwise)*	the inclination of the relative orbit to the plane of the sky, measured on the north side of the line of nodes with the secondary rotating in direct (counterclockwise) direction; **i** = 90° when orbit is perpendicular to line of sight
line of nodes	.	a line through the primary star and both nodes, common to both the apparent and relative orbits	
position angle of ascending node	Ω	position angle of ascending node measured counterclockwise from celestial north	.
argument of periastron	ω	.	the angle in the relative orbit from the ascending node side of the line of nodes to the periastron side of the major axis, measured in the direction of secondary rotation
Other Orbital Elements			

apastron	.	*the projection of this point*	the point in the relative orbit where the distance between the two stars is farthest and orbital speed is slowest
line of apsides	.	*the projection of this line*	a line in the relative orbit through the periastron, the primary star, and the apastron (= major axis of ellipse)
center	C	*the projection of this point*	the geometric center of the relative orbit, midway between the two foci
semiminor axis	b	*the projection of this line*	the distance (usually in arcseconds) in the relative orbit from the center C to the orbit, perpendicular to the semimajor axis

The orbital plane of the absolute orbit is almost never viewed in an orientation perpendicular to our line of sight from Earth. The orbital inclination (**i**) indicates the tilt of the relative orbit, which distorts both its apparent dimensions and eccentricity.

The inclination combines two different features of the relative orbit. First, it indicates the tilt of the plane of the relative and absolute orbits as an angle between the line of sight to Earth and the plane of the relative orbit, from 0° to 180° (diagram, below).



Second, the sign of the cosine of the inclination determines the direction of the secondary orbital motion as viewed from Earth: a direct (counterclockwise) orbit is coded as an angle between 0° and 90° (positive cosine), and a retrograde (clockwise) orbit is coded as an angle between 90° and 180° (negative cosine).

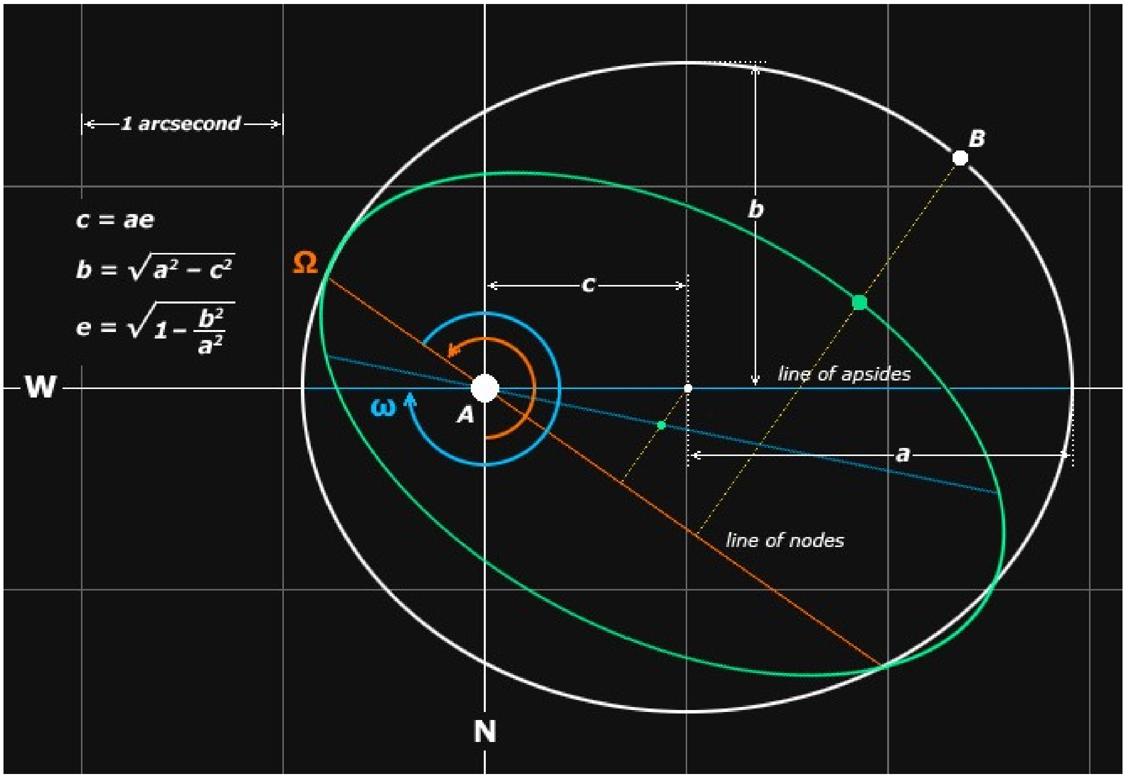
The line of nodes is the line formed by the intersection of the two planes of the true and apparent orbits, measured in counterclockwise direction from a line to the Earth's celestial north; it always passes through the primary (brighter or more massive) star.

The ascending node is the point on the line of nodes where the component star passes through the line of nodes and is moving away from Earth. In the great majority of binary stars where this cannot be determined due to an unmeasurably small orbital velocity, it is arbitrarily assigned to the position angle that is less than 180°. Thus, the inclination and ascending node in many cases represent an arbitrary rather than physical description of the binary system.

Note that the periastron rather than apastron is preferred as an orbital parameter because the relative orbital velocities of the two components at that point are at maximum. Either the radial velocity or the positional parameters (or both) will change most rapidly at that point, which usually minimizes error in the estimation of the time of periastron and therefore error in the predicted future relative positions of the components.

Diagramming a Double Star Relative Orbit

The Campbell elements can be used to diagram both the true and apparent orbits, and this is quite easy to do when working in Photoshop. The diagram below of iota Leonis provides a template.



1. Determine from the arcsecond scale of the semimajor axis the total system width and image scale. Use a large enough scale to minimize rounding errors. In the diagram, the semimajor axis equals 1.91", so the system width is about 4 arcseconds. The scale chosen for the example diagram is 120 pixels = 1 arcsecond.

2. Draw the cartesian x and y axes; the origin is the location of the system primary star.

3. Calculate **c** = **a**·**e** and convert to the image scale. In the example, **c** = 1.91·0.53 = 1.01 arcseconds, and 1.01·120 = 121 pixels. Measure and mark **c** on either the x or y axis of the plot, which becomes the *line of apsides* of the relative orbit.

4. Scale **a**, then measure and mark **a**+**c** along the line of apsides. In the example, the pixel scale of **a** = 1.91·120 = 229 pixels, so **a**+**c** = 229+121 = 350 pixels from the origin (or 229 pixels from **c**).

5. Calculate and scale **b** = $\sqrt{\mathbf{a}^2 - \mathbf{c}^2}$, then measure and mark vertically from **c**. In the example, **b** = $\sqrt{229^2 - 121^2}$ = 194 pixels.

6. Using the ellipical marquee tool while holding down the "Alt" key, click on **c** and stretch the marquee to create an ellipical area that exactly matches the marked distances **a** and **b**. Create a new layer, and fill the window; then use Modify —> Contract to reduce the selected area by the orbit line thickness you desire. Delete the window contents to create the relative orbit. Mark the orbit periastron, which is the end of line of apsides closer to the origin of the plot (the location of the primary star).

7. Create a new layer and draw a horizontal or vertical line, then rotate the line to correspond to the angle given as the argument of the periastron (**ω**). Note that if the angle of inclination is less than 90° then you must measure this angle **ω** in clockwise direction from the periastron. Move the rotated line so that it intersects the cartesian origin and the relative orbit. This rotated line is the *line of nodes,* and its intersection with the relative orbit at angle **ω** is the *ascending node.*

8. Copy the line of nodes layer, and rotate this line clockwise the PA of the line of nodes (**Ω**). This is the line of the primary star's right ascension in the apparent orbit on the celestial sphere.

9. Merge the orbit layer with the line of nodes layer, and copy this layer. Draw (copy) the line of apsides and point **c** onto this copied layer.

10. Rotate the copied orbit layer so that the line of nodes is exactly either horizontal or vertical, then use the Transform command to reduce the scale perpendicular to the line of nodes by a percentage equal to the cosine of the angle of inclination. In the example, the line of nodes is at 35° to the horizontal axis, so the copied layer was rotated counterclockwise by the same amount (–35°). Then **i** = 128°, and cos(**i**) = –0.62, so the copied orbit was reduced vertically to 62%.

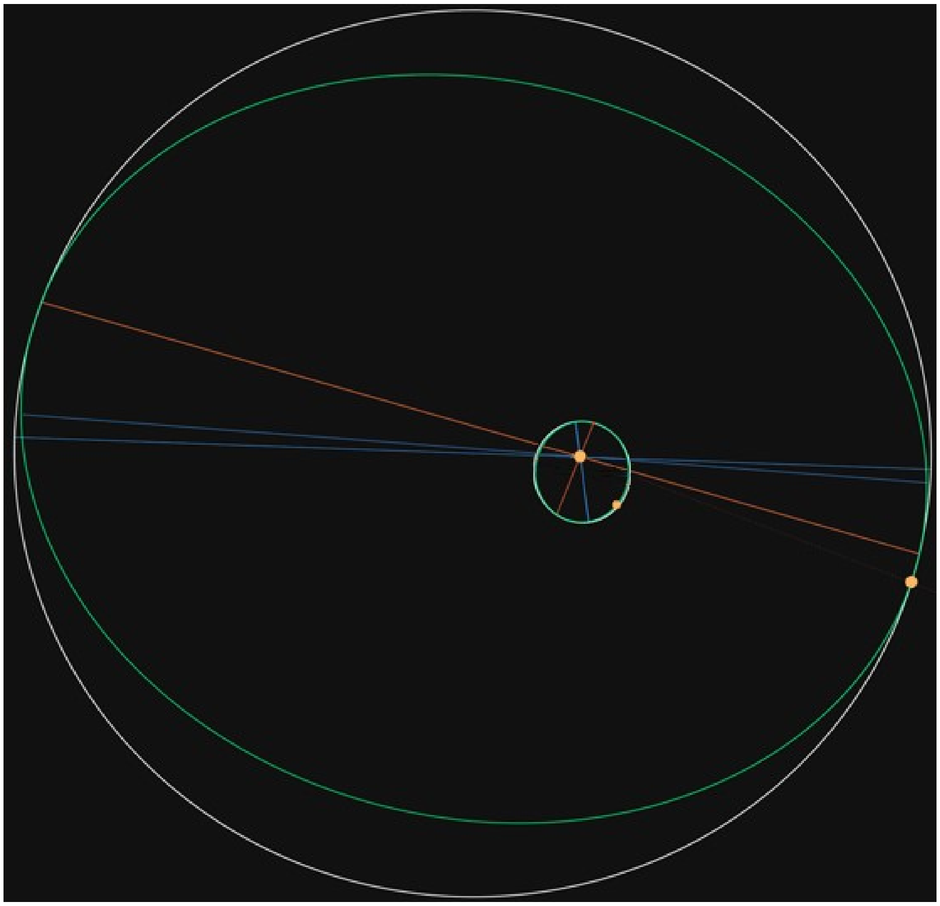
11. Rotate the copied orbit layer in the reverse direction so that the line of nodes is in its original orientation. Align the copied orbit so that the two orbits intersect at the line of nodes, and the intersection of the lines of apsides and nodes in the copied layer is at the origin of the plot. This is the apparent orbit.

12. Merge the orbits, and rotate them so that the line of right ascension is vertical, with north at the bottom.

13. If desired, use the catalog position angle for the secondary star to locate the star on the apparent orbit. Then use a line perpendicular to the line of nodes, and through the location of the secondary on the apparent orbit, to locate it on the relative orbit.

14. If necessary reduce and crop the canvas to the finished image size, and label elements as needed.

Multiple stars can be plotted in the same way, provided the orbital elements are available separately for hierarchical centers of mass: first A/B, then AB/C, then ABC/D, etc.



The diagram (above) of zeta Cancri (STF 1196) was created by first plotting the orbit of the AB pair, then the orbit for the AB/C "pair", rotating them both so that celestial north is at the bottom, then superimposing the primary of the AB pair on the "primary" focal point of the AB/C pair.

Further Reading

Multiple Star Orbits – an amusing group of animated double star orbits, helpful to visualize how complex gravity can be.