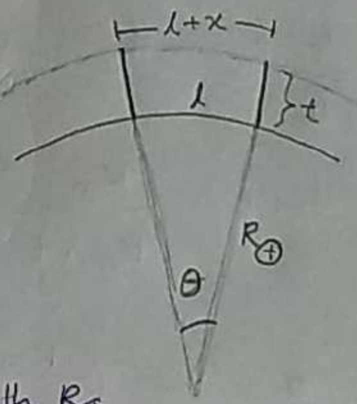
BDOAA-19

Set - α

1. Golden gate bridge



(a) ~~Radius of earth $R_\oplus$ is very very large compare to the lenght, l.~~

the bottom is part of the circle with the radius equal to the radis of earth. $R_\oplus$

the top is part of the circle with the radius equals to $(R_\oplus + t)$

$$\theta = \theta$$

$$\therefore \frac{l}{R_\oplus} = \frac{l+x}{R_\oplus + t} \qquad \boxed{1}$$

$$(R_\oplus + t)\, l = R_\oplus (l + x)$$

$$R_\oplus l + lt = R_\oplus l + R_\oplus x$$

$$lt = R_\oplus x$$

$$\therefore x = \frac{lt}{R_\oplus} \qquad \boxed{1}$$

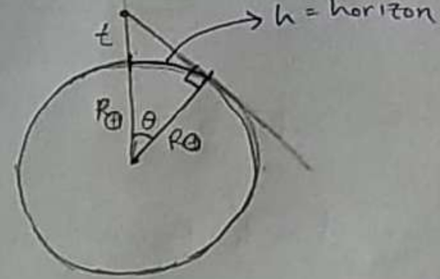
$$= \frac{1280 \times 230}{6.371 \times 10^6} = 0.0462 \text{ m}$$

$$= 4.62 \text{ cm} \qquad \boxed{1}$$

$a_\odot(1+e_\odot)$ $\quad 1.496\times10^{11}(1+0.017)$ —— $\boxed{1}$

$= 0.00915$ rad $= 1887''$ —— $\boxed{1}$

(b)



If the horizon create an angle θ in the centre of the earth.

$$\cos\theta = \frac{R_\oplus}{R_\oplus + t}$$ —— $\boxed{1}$

$$\therefore \theta = \cos^{-1}\left(\frac{R_\oplus}{R_\oplus + t}\right)$$

$$= 0.48^\circ \approx 8.497\times10^{-3} \text{ radian}$$ —— $\boxed{1}$

$\therefore$ the horizon, $h = R_\oplus\,\theta$

$$= 6.371\times10^{6} \times 8.497\times10^{-3} \text{ m}$$

$$= 54{,}134 \text{ m}$$ —— $\boxed{1}$

$\pi R_p^2 \quad \frac{L}{4\pi d^2}\,(1-$

$\therefore T_p = 4\sqrt{\,}$

2. $\boxed{\text{Habitable zone}}$

For a ~~star~~ planet of radius R_p at d distance away from the star.

the flux recieved. per second,

$$F_{reciere} = \frac{L_{\odot}}{4\pi d^2} \pi R_p^2 \qquad \text{———} \boxed{1}$$

Albedo is the percentage of energy reflected by the planet.

∴ Total absorbed Flux,

$$F_{absorbed} = \frac{L}{4\pi d^2} (1-\alpha) \pi R_p^2 \qquad \text{———} \boxed{1}$$

For a fast rotating ~~stable~~ star, the energy absorbed is equal to the energy radiated as luminosity.

$$E_{absorbed} = E_{emitted}$$

$$F_{absorbed} = L_{planet}$$

$$\cancel{\pi R_p^2} \frac{L}{4\pi d^2} (1-\alpha) = 4\pi \cancel{R_p^2} \sigma T_p^4$$

$$\therefore T_p = \sqrt[4]{\frac{L_{\odot} (1-\alpha)}{16 \pi \sigma d^2}}$$

$$d = \sqrt{\frac{L_{\odot} (1-\alpha)}{16 \pi \sigma T_p^4}} \qquad \text{———} \boxed{1}$$

For inner edge.

$$T_p = 100^\circ C = 373\ K$$

$$d_{in} = \sqrt{\frac{L_\odot (1-0.31)}{16\pi\sigma\ (373)^4}} \quad \boxed{0.5}$$

$$= 6.917 \times 10^{10}\ m$$

$$= 0.462\ AU \quad \boxed{0.5}$$

For outer edge,

$$T_p = 0^\circ C = 273\ K$$

$$d_{out} = \sqrt{\frac{L_\odot (1-0.31)}{16\pi\sigma\ (273)^4}} \quad \boxed{0.5}$$

$$= 1.291 \times 10^{11}\ m$$

$$= 0.863\ AU \quad \boxed{0.5}$$

(b) $d_{in} = a(1-e)$ $\qquad d_{out} = a(1+e)$ $\quad \boxed{0.5}$

$$\therefore \frac{d_{in}}{d_{out}} = \frac{1-e}{1+e} = \frac{1 - \frac{d_{in}}{d_{out}}}{1 + \frac{d_{in}}{d_{out}}} = \frac{d_{out} - d_{in}}{d_{out} + d_{in}} \quad \boxed{1}$$

$$\therefore e = \frac{d_{out} - d_{in}}{d_{out} + d_{in}}$$

$$\cancel{0.462 + 0.}$$

$$= \frac{0.863 - 0.462}{0.863 + 0.462}$$

$$= 0.302 \quad \boxed{0.5}$$

3. Ellipses and Eclipses

(a) Using small angle approximation, angular ~~radius~~, diameter,

$$\theta_{rad} = \frac{2R}{d}$$

The minimum diameter will be seen when the distance is maximum.

$$d_{max} = a_\odot (1 + e_\odot)$$

$$\therefore \theta_{min} = \frac{2R_\odot}{a_\odot(1+e_\odot)} = \frac{2 \times 7.00 \times 10^8}{1.496 \times 10^{11}(1+0.017)} \quad \text{—} \boxed{1}$$

$$= 0.00915 \text{ rad} = 1887'' \quad \text{—} \boxed{1}$$

$$d_{min} = a_\odot(1 - e_\odot) \quad \text{—} \boxed{1}$$

$$\therefore \theta_{max} = \frac{2R_\odot}{a_\odot(1-e_\odot)} = 1963'' \quad \text{—} \boxed{1}$$

(b) $$\theta_{min} = \frac{2R_m}{a_m(1+e_m)} = \frac{2 \times 1.7 \times 10^6}{3.84 \times 10^8 (1 + 0.055)} \quad \text{—} \boxed{1}$$

$$= 8.39 \times 10^{-3} \text{ rad} = 1731'' \quad \text{—} \boxed{1}$$

$$\theta_{max} = \frac{2R_m}{a_m(1-e_m)} = 9.36 \times 10^{-3} \text{ rad} = 1932'' \quad \text{—} \boxed{1+1}$$

(c) $$\text{Percentage} = \underbrace{\frac{A_{moon,\,min}}{A_{sun,\,max}}}_{\boxed{2}} = \underbrace{\frac{\frac{1}{4}\pi(\theta_{moon,\,min})^2}{\frac{1}{4}\pi(\theta_{sun,\,max})^2}}_{\boxed{1}} = \underbrace{\left(\frac{1731}{1963}\right)^2}_{\boxed{1}} = 77.76\%$$