BDOAA - 19

Regional Round

Junior - α - set

1. Starlight

(a) Stars twinkle due to the refraction of light in the different atmospheric layers which are changing in temperature and density. As the stars are too small in angular size

1. Starlight

(a) Key point :- 1. Refraction of starlight in atmospheric layers. — $\boxed{0.5}$

2. Stars are too small in size in the sky. — $\boxed{0.5}$

(b) $L_1 < L_2 \quad \therefore \frac{L_1}{L_2} < 1$

$$L \propto R^2 T^4 \Rightarrow \frac{L_1}{L_2} = \frac{R_1^2}{R_2^2} \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4 \quad [\because R_1 = R_2] \quad \text{—} \boxed{1}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{L_1}{L_2}\right)^{\frac{1}{4}}$$

$$\Rightarrow \frac{T_1}{T_2} < 1 \quad \boxed{\therefore T_2 > T_1} \quad \text{—} \boxed{1}$$

(c) The light spreads uniformly to the full inner surface of the sphere.

If the luminosity of a star is L, this L will spread uniformly to the full inner ~~radi~~ surface of a sphere of 1 km radius. — $\boxed{1}$

surface area $= 4\pi R^2$

" " at 1 km $= 4\pi (1000)^2$ m^2 or 4π km^2

$\therefore$ Light recieved in unit area.

$$\text{Flux, } F = \frac{L}{4\pi R^2} = \frac{L}{4\pi} \frac{\text{watt}}{\text{km}^2} \cancel{\text{or } \frac{L}{4\pi(1000)^2}}$$

(c) continues ....

→ If someone get $\frac{L}{4\pi R^2}$ or $\frac{L}{4\pi d^2}$ but don't put the value of R or d as 1 km → $\boxed{1.75}$

→ If they use the value ~~of~~ as 1 km → $\boxed{2}$

(d) They have same luminosity. $\therefore L_1 = L_2$

From (c) we get, Flux, $F = \frac{L}{4\pi d^2}$ [d = distance]

$$\therefore \frac{F_1}{F_2} = \frac{L_1}{4\pi d_1^2} \frac{4\pi d_2^2}{L_2} = \frac{L_1}{L_2}\left(\frac{d_2}{d_1}\right)^2 \quad \text{———} \boxed{1}$$

$$\frac{F_1}{F_2} = 10 \qquad \frac{L_1}{L_2} = 1 \qquad \therefore \left(\frac{d_2}{d_1}\right)^2 = 10$$

$$\Rightarrow \frac{d_2}{d_1} = \sqrt{10}$$

$$\therefore d_2 = \sqrt{10}\, d_1 \quad \text{———} \boxed{1}$$

$d_1 = 200$ AU $\quad \therefore d_2 \approx (\sqrt{10} \times 200) \approx 632.45$ AU

The time needed for the light to reach.

$$t_1 = \frac{d_1}{c} = \frac{200\text{ AU}}{3\times 10^8 \text{ m/s}} \qquad t_2 = \frac{d_2}{c} = \frac{632.45\text{ AU}}{3\times 10^8\text{ m/s}} \quad \text{———} \boxed{0.75}$$

$$= \frac{200 \times 1.496\times 10^{11}}{3\times 10^8}\text{ s} \qquad = \frac{632.45\times 1.496\times 10^{11}}{3\times 10^8}\text{ s}$$

$$\approx 99,733\text{ s} \qquad \approx 315381\text{ s}$$

——— $\boxed{0.25}$

2. Space Travel

(a) The star is at rest relative to sun.

$\therefore$ time $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{60 \text{ ly}}{\frac{4}{5} c}$ ——— $\boxed{1.5}$

$= 60 \times \frac{5}{4}$ year

$= 75$ year ——— $\boxed{0.5}$

(b) ~~$\dfrac{T_{rocket}}{T_{sun}} = \dfrac{1}{\sqrt{1 - v^2/c^2}}$~~

~~$= \dfrac{1}{\sqrt{1 - \left(\frac{4}{5}\right)^2}} = \dfrac{1}{\sqrt{1 - \frac{16}{25}}} = \dfrac{1}{\sqrt{\frac{25-16}{25}}} = \dfrac{1}{\sqrt{\frac{9}{25}}}$~~

~~$= \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$~~

~~$\therefore T_{rocket} = \frac{5}{3} T_{sun}$~~

$\dfrac{T_{rocket}}{T_{sun}} = \sqrt{1 - \dfrac{v^2}{c^2}}$ ——— $\boxed{0.5}$

$= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\therefore T_{rocket} = \frac{3}{5} T_{sun}$ ——— $\boxed{1.5}$

(c) From (a) $T_{star} = 75$ year

$\therefore T_{rocket} = T_{star} \frac{3}{5}$ ——— $\boxed{1}$

$= 75 \times \frac{3}{5} = 45$ year ——— $\boxed{1}$

(d) The star is at rest relative to the sun

∴ Time in star clock will be same as time ~~∮~~ in sun clock — $\boxed{1.5}$

∴ $T_{sun} = T_{star} = 75$ year — $\boxed{0.5}$

(e) The signal reached the star at the same time of the spaceship.

∴ T_{star} when ~~light~~ signal recieve $= 75$ year.

Distance of sun from the star $= 60$ light year.

∴ The actual explosion happens, when $T_{star} = 75 - 60$
$= 15$ year — $\boxed{1}$

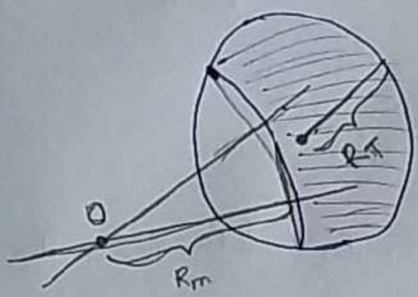
Time at explosion,

~~T_{star}~~ $T_{sun} = T_{star} = 15$ year

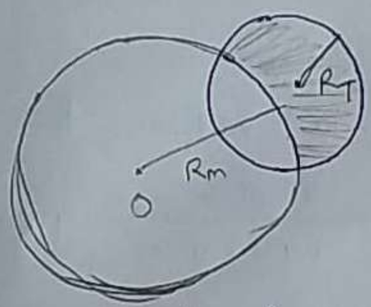
$T_{rocket} = \frac{3}{5} T_{star} = \frac{3}{5} \times 15 = 9$ year — $\boxed{1}$

3 Moon Picture

(a) ~~First approach: Using ruler~~



Determine the center of moon, O using two chord (জ্যা) and their midpoint perpendicular line (লম্ব সমদ্বিখণ্ডক).



or, using compass and trial and error method, determine O. —— $\boxed{1}$

Measure R_m and R_T, radius of the moon circle and the telescope circle and take their ratio: $\frac{R_m}{R_T} \approx 2$

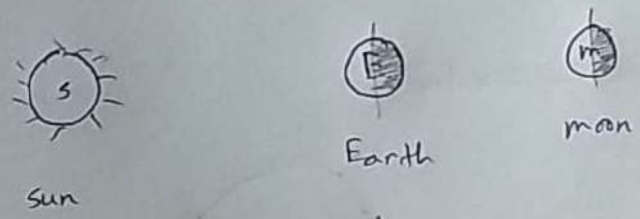
$R_T = \frac{1}{2} 16' = 8'$

$\therefore R_m = \left(\text{ratio of } \frac{R_m}{R_t}\right) \times 8' \approx 16' \approx 4{\cdot}65 \times 10^{-3}$ radian —— $\boxed{1}$

Actual radius $= \left(R_m \text{ in radian}\right) \times$ distance

$= 4{\cdot}65 \times 10^{-3} \times 3{\cdot}844 \times 10^{8} \approx 1790$ km —— $\boxed{1}$

(b)



$\boxed{2}$

(c) Approch 1:

Using compass or ruler determine the centre and radius of the moon circle and sun circle & and take their ratio just like (a).

~~Now we know the~~

Approch 2:

$$R_m = 16' \approx 4.65 \times 10^{-3} \text{ radian} \quad \text{from (a)}$$

$$R_s = \frac{\text{Radius of sun}}{\text{distance of sun}} = \frac{7 \times 10^{8} \text{ m}}{1.496 \times 10^{11} \text{ m}} = 4.679 \times 10^{-3} \text{ radian}$$

$$\therefore \frac{R_m}{R_s} = \frac{4.65 \times 10^{-3}}{4.679 \times 10^{-3}} = 0.9938$$

$\boxed{2.5}$

$$\text{The covered area} = \frac{A_{moon}}{A_{sun}}$$

$$\frac{A_{moon}}{A_{sun}} = \frac{\pi R_m^2}{\pi R_s^2} = \left(\frac{R_m}{R_s}\right)^2 = (0.9938)^2$$

$\boxed{1}$

$$= 0.9876 = \boxed{98.76\%}$$

$\boxed{0.5}$