4^{th} Bangladesh Olympiad on Astronomy and Astrophysics

Sample Problems 2021 - **Solutions**

Note: These question are above the usual standard for first round exam of BDOAA, so don't get frustrated. This questions are set to give you an idea on what to expect. In this solutions paper we will try to discuss all the necessary things needed to understand these topics, so you can think this paper as a study note!

1 Meteor Struck

It is said that it is rare to be hit by a meteorite on the Earth. Let us try to investigate how rare it is on the Moon. The radius of moon, $R_{☾} = 1,737.1$ km.

a. Assuming that it is a perfect sphere, calculate its surface area.
   About 2700 kg of cosmic material falls onto the Moon daily. Most of it consists of microscopic particles and dust. Assume that all of them had the size of air rifle bullets with mass 0.500 g and that they cover the surface of the Moon homogeneously.

b. Calculate the surface area on which (on average) exactly one bullet galls per day. Round the result to 3 significant digits.

c. The Apollo 11 mission explored about $750\,m^2$ of the Moon's surface. How many days on average should we wait until a meteorite of the size of this bullet falls onto this area?.

d. What are the odds that such a body will hit an astronaut during a day on the Moon outside the landing Module.

Solution Prerequisite Knowledge

- Celestial Mechanics (Gravitation),
- Significant Digits and Rounding estimates
  `https://faculty.virginia.edu/ASTR3130/lectures/error/significant.html`
- Geometry of Objects,
- Mass Flux.

Solutions:

a. Given that, Radius of the moon = 1737.1 Km
Surface area of the moon,

$$= 4\pi R_{☾}^2$$
$$= 4 \cdot 3.1415 \cdot (1737100)^2$$
$$= 3.792 \times 10^{13}\, m^2$$

Notice that we rounded the answer to three significant figures because we have to round answer in b. to three significant figures. It wont matter if you round it to more than 3 significant figures but if you round this answer to less than three significant figure, the accuracy will be lost in the next answer.

b. Number of bullets,[Don't forget to convert the unit!]

$$= \frac{2700}{0.0005} = 5.4 \times 10^6$$

Area in which a single bullet falls,

$$= \frac{3.792 \times 10^{13}}{5.4 \times 10^6} = 7.022 \times 10^6\, m^2$$

c. Number of bullets that fall in $750\, m^2$ in a day

$$= \frac{750}{7.022 \times 10^6} = 1.068 \times 10^{-4}$$

Number of days required for a bullet to fall

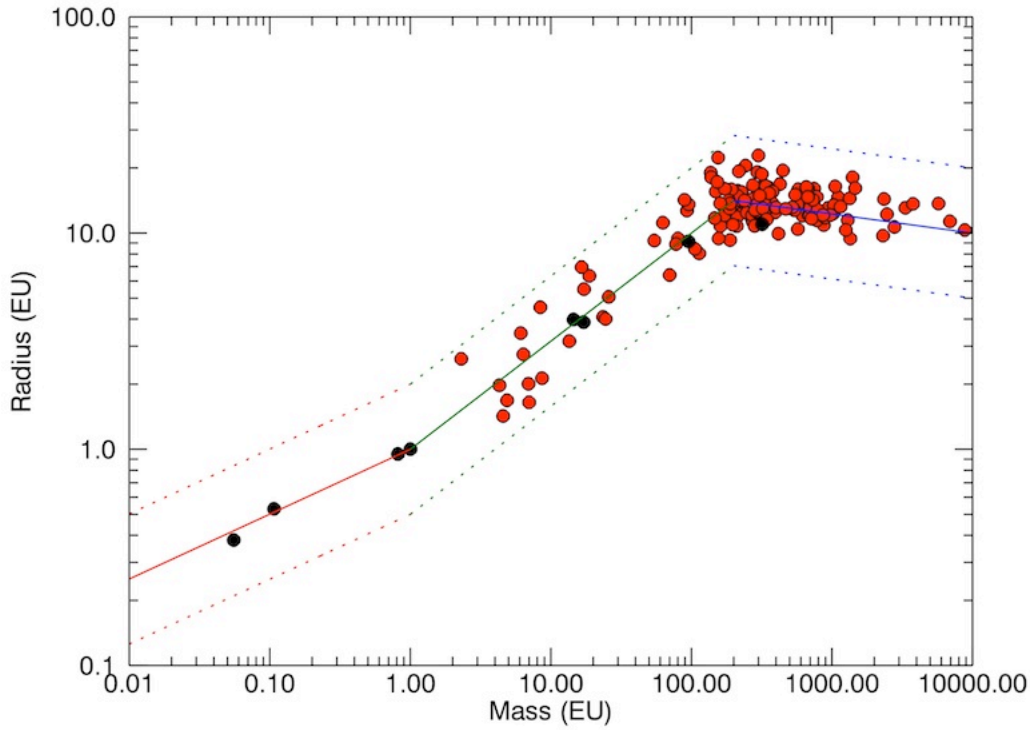
$$= \frac{1}{1.068 \times 10^{-4}} = 9363.296\, days$$

d. For this problem, we have to approximate the cross section area of an astronaut. We assume that an astronaut has a circular cross section.we need to find the radius of this cross section. Notice that the distance from one shoulder to your other shoulder is a bit bigger than a regular ruler that is a bit bigger than 0.3 m. Since astronauts have to wear space suits, we can assume the diameter of the cross section to be double than that. So we are assuming the diameter to be 0.6m, Cross section of an astronaut, $= \pi r^2 = 3.1416 \times (\frac{0.6}{2})^2 = 0.283\, m^2$.
In one day, 1 bullet falls on an area of $7.022 \times 10^6 m^2$
So the odds that 1 bullet will hit an astronaut in a day

$$= \frac{0.283}{7.022 \times 10^6} = 4.030 \times 10^{-8}$$

2 Exoplanetary Scientist

Fahim worked on exoplanets during 2020 where he was curious about the evolution of the exoplanetary atmosphere. From his research he found this graph—



Empirical mass-radius relation for exoplanets. The dotted lines show two and half times the predicted value, all fall within this maximum error boundary. The data is from confirmed exoplanets with mass and radius. EU = Earth units.

a. From the graph predict 3 separate equations for mass radius relation of these data points.

b. If the radius of the Earth is 6,371 km calculate the mass of the Earth, and the kinetic energy and pressure involved when the Earth gets hit by a meteorite of mass 0.001 EU, density $3400\,\mathrm{kg/m^3}$.

Solution Prerequisite Knowledge

- **Linearization of Graphs**: When two variables are plotted and the resulting graph is non-linear(power,exponential,or sinusoidal,for example), it is difficult to determine the functional relationship between the two variables from the shape of the curve. However, there are several techniques that can be used to turn a non-linear graph into a linear one, and they are discussed below.
  An equation of the form:
  $$y = mx^A$$
  is called a power relation. This type of relation occurs frequently in physics and graphically yields a curve when y is plotted against x. However,it is very difficult to determine the exact value of the power A simply by looking at the shape of the curve. A simple technique, called the log-log method, solves this problem by showing whether a power relation actually holds, and if so, by giving the numerical value of A. As the name implies, this method involves taking log of both sides of the equation as follows
  $$log\, y = A\, logx + logm$$
  By comparing Equation to $y = mx + b$, we see that a graph of $logy$ on the vertical axis versus $logx$ on the horizontal axis yields a straight line with slope A. Once A is known, we can then plot a graph of y versus x^A ("Change of Variable" method) which yields a straight line whose slope is the value of the constant m.

- Different types of Graph– log-log
  `https://undergroundmathematics.org/exp-and-log/plotting-planets/solution`

- Work, Energy, and Power- See your general physics book!

- Exoplanet Structure Basic Idea- arXiv:1312.3323v1
  `https://arxiv.org/abs/1312.3323`

Solutions:

a. For the first problem, with a ruler draw 3 separate line and find their slope. Remember that the markings for axis are powers of 10 which makes it a log-log graph. Using the equation of previously measured straight lines convert them to their actual in exponential form [effectively reversing the Linearization method]
The equation for the radius,

$$r = \begin{cases} m^{0.3} & m < 1 \\ m^{0.5} & 1 \leq m < 200 \\ (20.6)m^{-0.0886} & m \geq 200 \end{cases}$$

b. For this question we assume that, $V_\infty = 0$
Suppose in a general context, $D_{met} = 10$ km (You should find the exact diameter using equations derived from a.), Kinetic Energy,

$$KE = MV^2/2$$

A 10 km meteoroid would experience negligible amount of air resistance when it enters the Earth atmosphere. Therefore its speed would be the free fall velocity,

$$V = (2GM_\oplus/R_\oplus)^{1/2} = 11\, kms^{-1}$$

With a radius of 5 km (ie half of 10 km), its mass is

$$M = 4\pi\rho R^3/3 \simeq 1.8 \times 10^{15}\, kg$$

Therefore its $KE = 1.08 \times 10^{23}$ Joules which is equivalent to 2.5×10^7 Mega tons of TNT.
Consider a layer in the meteoroid with an area A and thickness Δd so its mass is

$$M = \rho A \Delta d$$

Then this layer reaches the ground, its speed must decrease from V to 0 during its travel time which is $\Delta d/V$. Its acceleration would be $g = V^2/\Delta d$. The pressure P experience by the meteoroid is the force

$$P = Mg/A$$

where A is the area. Thus

$$P = Mg/A = (\rho A \Delta d)(V^2/\Delta d)/A = \rho V^2$$

In this case $P = 4.114 \times 10^{11}$ Pascals which is 4×10^6 atmospheric pressure.

3 Celestial Mechanics

A minor planet revolves around the Sun. This minor planet is observed in the same position every two years, under constant conditions. Throughout these two years, the maximum and minimum difference in magnitude observed from Earth is 8. Consider the minor planet to be smooth, spherical, with constant surface albedo and that it can be observed during the day as well. You may take Earth's orbit to be circular.

a. How many revolutions per 2 years does the minor planet make if its orbit never comes inside Earth's orbit?

b. Explain why the minor planet cannot have an orbital period of 1 year if the minor planet's orbit is elliptical or if the minor planet's orbit is circular.

c. Using Kepler's 3rd law, deduce the minor planet's semi-major axis, a.

d. Sketch a diagram showing the relative positions of the Sun, Earth and the minor planet when the minor planet is at maximum and minimum magnitude. Label these points (1) and (2) respectively.

e. Find the minor planet's minimum eccentricity e. The observed magnitude m is given by

$$m = H + 2.5\,log\left(\frac{d^2r^2}{p(\chi)}\right),\ 0 \leq \chi \leq 1.$$

where d is Sun-object distance, r is Earth-object distance, $p(\chi)$ is the phase integral which represents how much light is reflected that depends on the phase angle. H is the apparent magnitude of an object when viewed at exactly 1 AU with a full phase integral. Note: If you are unable to derive the semi-major axis in part c., you may use the value 2.0 AU for this part.

Solution Prerequisite Brief notes

- **Flux, Magnitude, and Albedo for Blackbody**
  Brightness of Solar System bodies: The brightness of planets and satellites changes over time. It is influenced by the following factors:
  - change in the distance l of the planet from the Sun, accompanied by a change in sunlight on the planet itself;
  - change in the distance r of the planet from the Earth, which also changes the brightness inversely proportional to the square of this distance;
  - phase of the planet - with a change in the angle χ between the Sun and the observer from the center of the planet changes the area of the illuminated part of the disk and the brightness of its surface relative to the observer.
  - uneven color and landscape of the planet, physical changes in its atmosphere.
  - More details: `https://en.wikipedia.org/wiki/Absolute_magnitude`

Magnitude Scale: The magnitude scale is fixed so that two stars which has a flux ratio of 100 differ by 5 magnitudes. For example, a 1^{st} magnitude star has a flux 100 times larger than a 6^{th} magnitude star, which itself has a flux 100 times larger than an 11^{th} magnitude star, and so on. Thus a 1^{st} magnitude star has a flux $100 \times 100 = 10,000$ times larger than an 11^{th} magnitude star. In general, a star n magnitudes brighter than another has flux $10^{n/5}$ times larger. In other words, if star 1 has flux f_1 and magnitude m_1, and star two has flux f_2 and magnitude m_2, then the flux ratio is given by.

$$\frac{f_1}{f_2} = 100^{(m_2-m_1)/5}$$

Since $100 = 10^2$ this converts to:

$$\frac{f_1}{f_2} = 10^{2(m_2-m_1)/5} = 10^{0.4(m_2-m_1)}$$

A difference of 1 magnitude corresponds to a flux ratio of $10^{2/5} = 2.512$. Notice that if m_2 is bigger than m_1, then f_2 is smaller than f_1 (the "backwards" character of the magnitude system). Another way of expressing this is to take the logarithm of both sides of the equation:

$$log\left(\frac{f_1}{f_2}\right) = log\left(10^{0.4(m_2-m_2)}\right) = 0.4(m_2 - m_1)$$

which is usually written as,

$$m_1 - m_2 = -2.5\, log\left(\frac{f_1}{f_2}\right)$$

Caution

- Astronomical Magnitude কে সাধারণত সংখ্যার সাথে সূচক আকারে লেখা হয় যেমন 1^m (যেন m দিয়ে প্রকাশিত অন্যান্য এককের সাথে গোলামাল না হয়ে যায়!
- বাংলাতে সাধারণত অলিম্পিয়াডের প্রশ্ন করা হলে Apparent Brightness (B) কে আপাত উজ্জ্বলতা এবং Apparent Magnitude (lower case m) কে আপাত ঔজ্জ্বল্য/ উজ্জ্বলতার মাত্রা/মান বলব। একই ভাবে Absolute Magnitude (upper case M) কে পরম ঔজ্জ্বল্য /উজ্জ্বলতার মাত্রা বলব।

- Orbits (Elliptical, parabolic, and circular),
  Reading: `https://phys.libretexts.org/Bookshelves/Astronomy__Cosmology/Book%3A_Celestial_Mechanics_(Tatum)/02%3A_Conic_Sections`

- Kepler's Laws.
  Reading: `https://phys.libretexts.org/Bookshelves/Astronomy__Cosmology/Book%3A_Celestial_Mechanics_(Tatum)/09%3A_The_Two_Body_Problem_in_Two_Dimensions`

Solutions:

a. 1 revolution per 2 years.

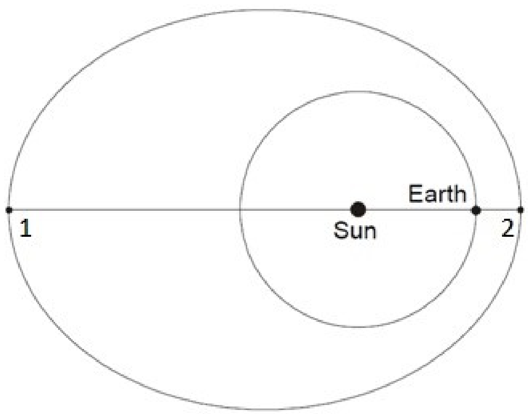
b. If the orbit is elliptical, the minor planet's orbit would intersect Earth's orbit because the semi-minor axis would be less than 1 AU.

c. If the orbit is circular the minor planet would always be in the same position relative to Earth and its brightness would never change.

$$T^2 = a^3 \tag{1}$$

$$a_{minor\,planet} = 2^{\frac{2}{3}} = 1.587AU \tag{2}$$

d. Here is an approximate diagram



e. For this particular case the phase, $\chi = 1$, so our equation simplifies like,

$$m = H + 5\,log\,(d) + 5\,log\,r$$

d is Earth-asteroid distance, r is Sun-asteroid distance. Subscript 1 refers to position 1:

$$d_1 = a(1-e) - a_0$$
$$r_1 = a(1-e)$$
$$d_2 = a(1+e) + a_0$$
$$r_2 = a(1+e)$$

$$5\,log(d_2) + 5\,log(r_2) - 5\,log(_1) - 5\,log(r_1) = 8$$

$$\frac{d_2 r_2}{d_1 r_1} = 10^{1.6} = 39.8 = K = \frac{[a(1+e)+a_0]\,a(1+e)}{[a(1-e)-a_0]\,a(1-e)}; a_0 = 1AU$$

$$e = \frac{2(K+1)a - (K-1)a_0 \pm \sqrt{(K-1)^2 a_0^2 + 16Ka^2}}{2a(K-1)} = 0.284$$

4 Astronomy on Mars

Elon Musk has sent Neha on mars to navigate Martian sky. Neha noticed that the Martian north celestial pole is located in Cygnus and has coordinates– **Declination**, $\delta_M = +52°53.0'$ and **Right Ascension**, $\alpha_M = 21^h10^m42^s$.

a. Mark the location of these coordinates on the star map provided. Consider this map as similar to Declination ≡ Latitude and R.A.≡ Longitude.

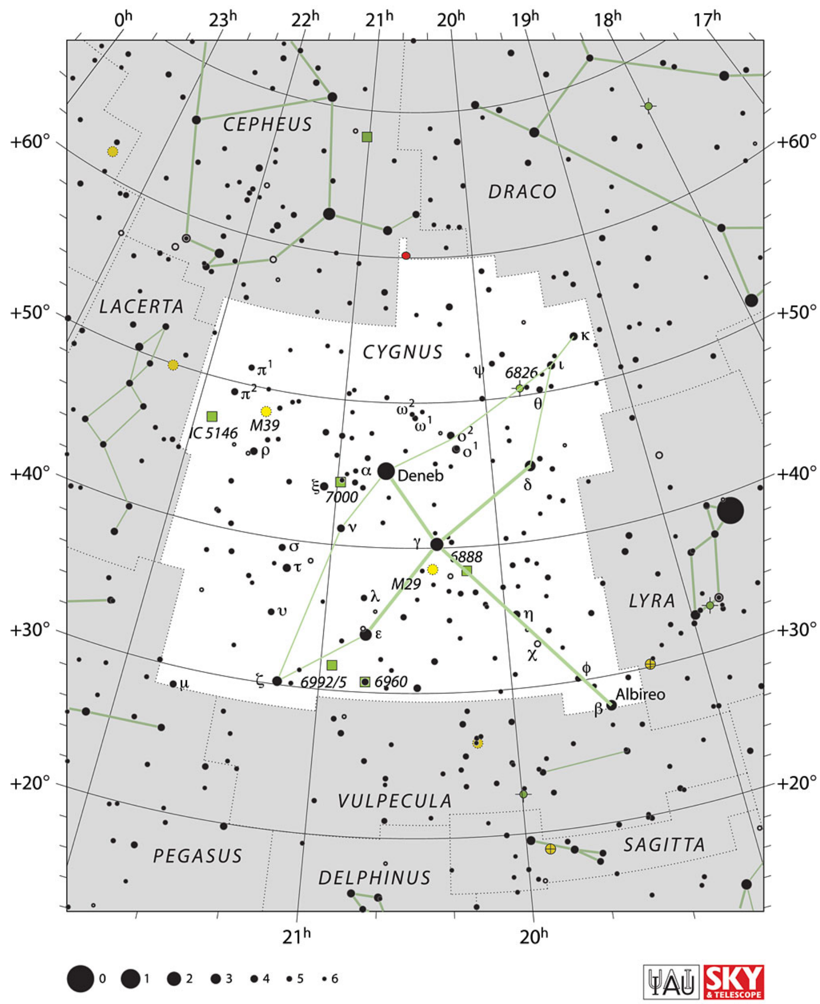


Image taken from: `https://www.iau.org/public/images/detail/cyg/`

b. The constellation Cygnus can be viewed on the North-western side of our Rajshahi Sky. Consider that Cygnus never sets below the horizon for a certain location on Mars' surface and it is also viewed from the North. Find the latitude, ϕ_M of that location on Mars surface

Solution Prerequisite Knowledge

- Constellations and Stars
https://bdoaa.org/%e0%a6%a4%e0%a6%be%e0%a6%b0%e0%a6%be%e0%a6%9a%e0%a6%bf%e0%a6%a4%e0%a7%8d%e0%a6%b0-%e0%a6%8f%e0%a6%ac%e0%a6%82-%e0%a6%86%e0%a6%95%e0%a6%be%e0%a6%b6-%e0%a6%9a%e0%a7%87%e0%a6%a8%e0%a6%be/
- Astronomical Spherical Coordinate System,
https://phys.libretexts.org/Bookshelves/Astronomy__Cosmology/Book%3A_Celestial_Mechanics_(Tatum)/06%3A_The_Celestial_Sphere
- Geometry of Celestial Sphere.
https://www.youtube.com/watch?v=_NVjS2Ue43Q

Solutions:

a. The Vertical Axis acts as declination, as it is for a spherical projection the upper portion of the Map will be distorted. The Horizontal axis similarly acts as right ascension.

b. Circumpolar Stars: An object is said to circumpolar if it is so close to one of the celestial poles that it never sinks below the observer's horizon. As the night sky rotates around the celestial poles (labelled NCP and SCP in the diagram to the right), most objects rise above the eastern horizon, and set along the western horizon.
However, objects which are close to the celestial poles either remain permanently above the horizon, if they are in the same hemisphere as the observer, or never rise above the horizon at all, if they are in the opposite hemisphere to the observer.For a star to be circumpolar, the star must, at the very least, come above the horizon.

$$0 \leq a \leq \frac{\pi}{2}$$

In the celestial sphere coordinate system, c can be found to be

$$c = \frac{\pi}{2} - \delta$$

In the horizon coordinate system, c can be found to be

$$c = \phi - a_{min}$$

Therefore,

$$\phi - a_{min} = \frac{\pi}{2} - \delta$$

$$\phi + \delta = \frac{\pi}{2} + a_{min}$$

a_{min} can be set to 0 because it is the lower bound for circumpolarity.

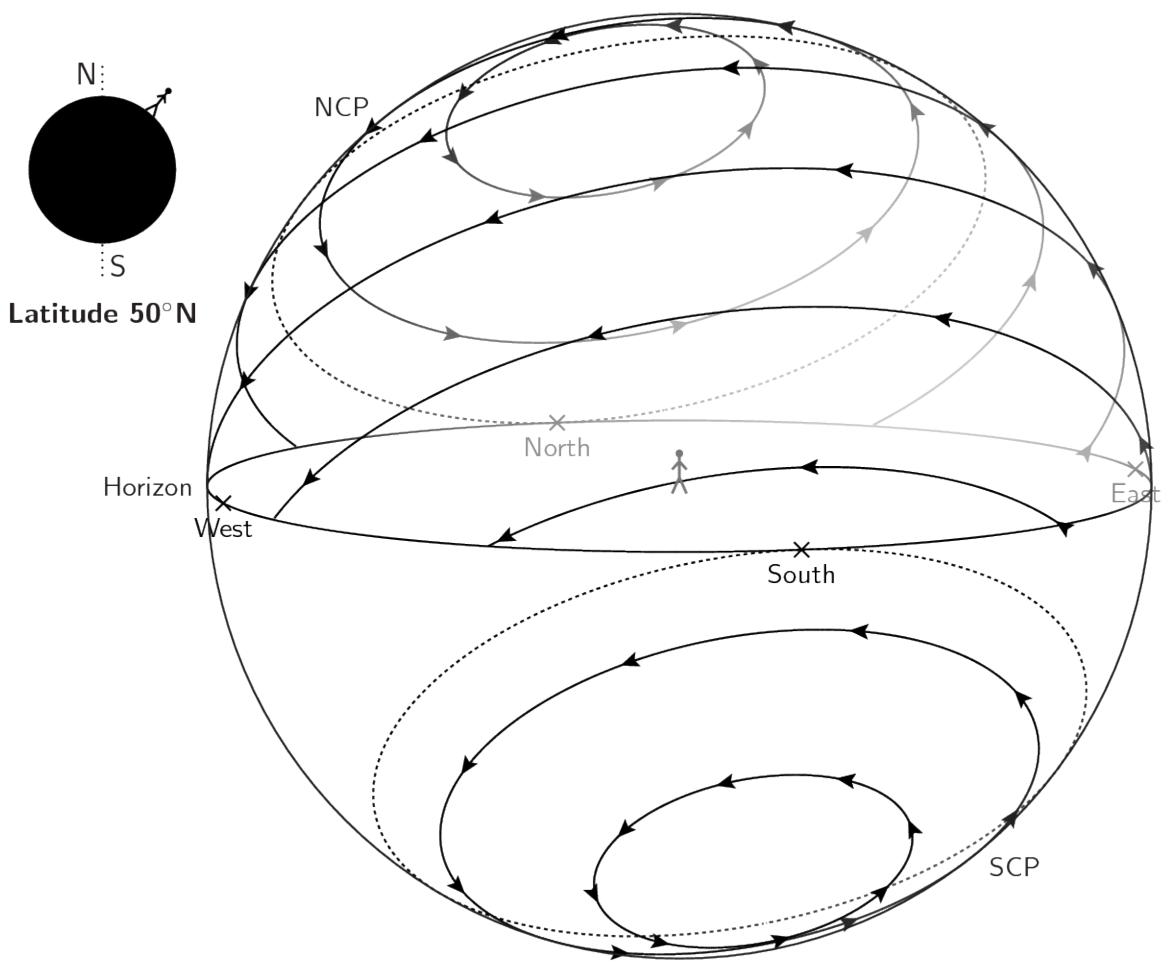
$$\phi + \delta = \frac{\pi}{2}$$

δ must be in the cone made by the star's rotation about its declination angle. Therefore

$$\delta + \phi \geq \frac{\pi}{2}$$

Now to find the martian latitude one must find what will be the Martian declination of the same star/position viewed from Earth. One way to find this comparing NCP of both planets. Given that,
Martian NCP on Earth $\equiv \delta_{M\oplus} = +52°53.0' \equiv \delta_M = 90°$ on Mars. So any star will be inclined by an angle, $90° - 52°53.0'$.
So we get the final equation to be,

$$\phi_{\text{Mars}} \geq \frac{\pi}{2} - (90° - 52°53.0')$$



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