# $4^{\text{th}}$ Bangladesh Olympiad on Astronomy and Astrophysics

First Round 2021- Set A **Solutions**

April 30, 2021

**Instructions for the Candidate:**

- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Be sure to calculate the final answer in the appropriate units asked in the question.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.
- The exam duration is 1 hour and you'll have extra 15 minutes to compile the answers and submit the PDF into the designated portal.

Table 1: **Useful Constants and Formulas**

| | | | |
|---|---|---|---|
| Mass of the Sun | $M_\odot$ | $\approx$ | $1.989 \times 10^{30}$ kg |
| Mass of the Earth | $M_\oplus$ | $\approx$ | $5.972 \times 10^{24}$ kg |
| Mass of the Moon | $M_☾$ | $\approx$ | $7.347 \times 10^{22}$ kg |
| Radius of the Earth | $R_\oplus$ | $\approx$ | $6.371 \times 10^{6}$ m |
| Radius of the Sun | $R_\odot$ | $\approx$ | $6.955 \times 10^{8}$ m |
| Speed of light | $c$ | $\approx$ | $2.99 \times 10^{8}$ m |
| Astronomical Unit(AU) | $a_\oplus$ | $\approx$ | $1.496 \times 10^{11}$ m |
| Solar Luminosity | $L_\odot$ | $\approx$ | $3.826 \times 10^{26}$ W |
| Gravitational Constant | $G$ | $\approx$ | $6.674 \times 10^{-11}$ Nm$^2$kg$^{-2}$ |
| 1 parsec | $1\,pc$ | $=$ | $3.986 \times 10^{16}$ m |
| Stefan's constant | $\sigma$ | $=$ | $5.670 \times 10^{-8}$ Wm$^2$K$^{-4}$ |

# 1 MCQ

Answer the following multiple choice questions. Each question contains **1 mark**.

a. Three friends are arguing over the positions of sunrise and sunset at different latitudes and seasons.
Turja: The sun rises due east and sets due west. Differences in latitude only changes the angle of the sun's path in the sky.
Ankon: No, the point of sunrise and sunset can be anywhere between 0 degree and 180 degree from North on the East and West sides respectively depending on latitude and season.
Serat: Both of you are wrong. While it is true the point of sunrise and sunset varies according to latitude and season, it can only vary between 30 degrees and 120 degrees from due north.
Who is correct?

   i. Turja
   ii. Ankon
   iii. Serat
   iv. None of them

> Answer **ii.**
>
> Option **i.** is wrong because the sun only rises and sets due East and West at equinox. For option **iii.**, 30 degrees and 120 degrees azimuth are random values with no physical basis whatsoever. Options **iv.** is non-answer to begin with.
>
> Option **ii**: Consider the scenario of the sun at summer solstice, i.e. at its most northerly declination of 23.4 deg. The diagram in Porblem 2 shows the path of the sun at the north hemisphere, with the arrow pointing towards the north celestial pole. N Moving towards more southern latitudes, at about 66.6 deg N, the path of the sun tilts such that it touches the horizon at a single point, which is in the direction of due North. Moving further south, the path of the sun crosses the horizon at two points on the northeast and northwest. At the equator, the sun rises 23.4 deg north of east and sets 23.4 deg north of west. Moving south of the equator, the scenario repeats, and the point of sunrise and sunset slowly converges towards the north, and you can see that the length of the day gets shorter and shorter until it reaches eternal night beyond 66.6 deg South. Thus at summer solstice the azimuth of sunrise ranges between due North and 23.4 deg north of east across latitudes. The inverse is true at winter solstice, where the azimuth of sunrise ranges between due south and 23.4 deg south of east. The sun rises between 23.4 deg north and 23.4 deg south of east in the period between solstices at latitudes nearer to the equator.

b. How far from the Earth would the Sun have to be moved so that its apparent angular diameter would be 1 arc second?

i. 1900 AU
ii. 3.26 AU
iii. 206265 AU
iv. 10 AU

Answer **i.**

**First convert arcsec to radians:** 1 arcsec = $=(180 \times 60 \times 60) = 4{:}8 \times 10^{-6}$ radians.

$$D = \frac{2R_\odot}{} = \frac{2R_\odot}{4{:}8 \times 10^{-6}} = 4{:}1 \times 10^{15}\, R_\odot = \boxed{1900\,\mathrm{AU}}$$

c. A Dyson sphere is a hypothetical megastructure that completely encompasses a star and captures a large percentage of its power output. Suppose we observe a Dyson sphere with a radius of 200 AU and temperature of 3400 K. At what wavelength of EM radiation does it emit most intensely? Assume that the sphere is a black body.

i. 760 nm
ii. 820 nm
iii. 850 nm
iv. 920 nm

Answer **iii.**

Just use Wien's displacement Law here,

$$_{\max} = \frac{2{:}897 \times 10^{-3}}{3400} = 851\mathrm{nm} \approx 850\,\mathrm{nm}$$

d. An Earth-like planet is orbiting its star at 1 AU. If the star is determined to have a mass double that of the Sun, which of the following is the orbital period of the planet in Earth years?

i. 1.6 y
ii. 0.7 y
iii. 0.5 y
iv. 1.2 y

Answer **ii**.
Observe Kepler'sThird Law implies that for constant radius:

$$T^2 \propto \frac{1}{M}$$

$$\frac{T}{T_{\oplus-\odot}}^{2} = \frac{M_{\oplus-\odot}}{M}$$

*$\text{T}_{\oplus-\odot} = 1$ y; $\frac{M_{\oplus-\odot}}{M}$= -0.5,

$$T^2 = 0{:}5T = \sqrt{2}\text{y}$$

e. Procyon is amongst the brightest stars in the night sky, and is the brightest star in the constellation, Canis Minor. It has an absolute magnitude of 2.6. What is the distance of a similar star, with apparent magnitude 13, from Earth?
We know that,

$$M_1 - M_2 = -2{:}5\, log \quad \frac{d_2^2}{d_1^2}$$

i. 600 pc
ii. 5 pc
iii. 10 pc
iv. 1.2 kpc

Answer **iv.**

Plug into the distance modulus formula and solve the equation:

$$m - M = 5 \log \quad \frac{d}{10\,\text{pc}}$$

$$13 - 2{:}6 = 5 \log \quad \frac{d}{10\,\text{pc}}$$

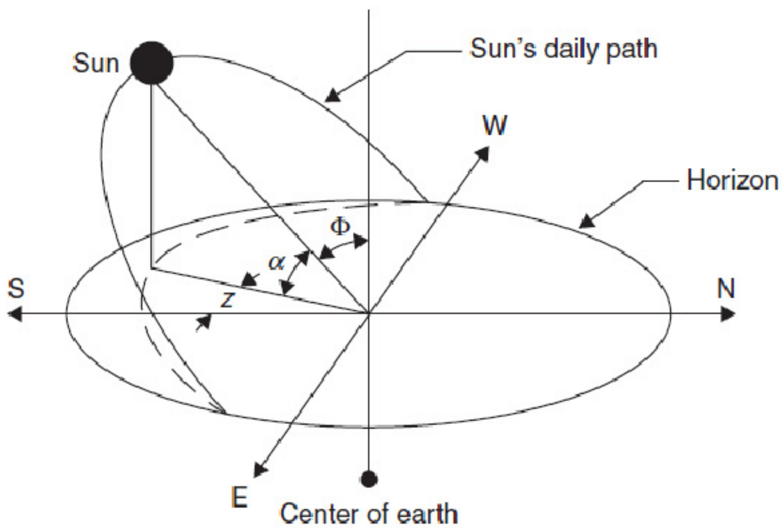
$$\frac{d}{10\,\text{pc}} = 10^{\frac{10:4}{5}}$$

$$d \approx 1200\,\text{pc}$$

# 2 Sun with it's troubled path!

Due to Earth's obliquity, $\ = 23.5°$, for an observer on Earth the Sun seem to change it's position on the Sky with time. In this question we'll explore how the position changes throughout the year.

a. What is the ecliptic latitude and longitude of the Sun on May $1^{st}$. [2]




Apparent motion of the Sun from Northern hemisphere

b. Calculate the Sun's right ascension, $\ _{\odot}$ and declination, $\ _{\odot}$ for this date. [1.5+1.5]

c. An Observer is situated in latitude $\ = 25°$ N, longitude $\ = 0°$ W. What is the interval (in solar time) between sunrise and sunset for this observer on this date? Determine the Greenwich mean times of sunset and sunrise. We know that,

$$\cos 90° = \cos(90° - \ ) \cos(90° - \ ) + \sin(90° - \ ) \sin(90° - \ )$$

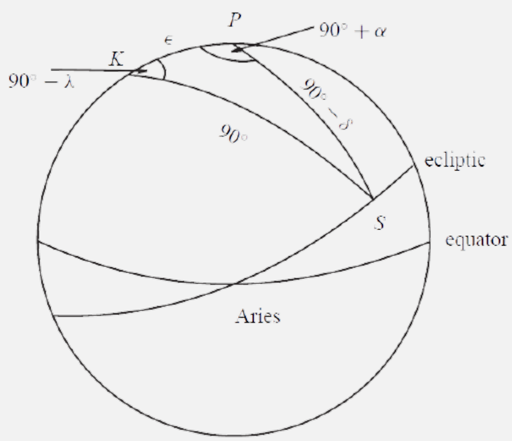
You should draw a celestial diagram to try to understand the situation. [3]

a. The Sun always lies on the ecliptic and so its ecliptic latitude is always zero. [1]
The ecliptic longitude   is zero on March 21st when the Sun is at first point of aries. It increases uniformly by 360° in one year. May 1st is 41 days after March 21st and so we may calculate,

$$= 41 \times \frac{360^\circ}{365{:}25} = 40^\circ{:}4$$

[1]

b. Consider the diagram as shown, in which S is the Sun (on May 1st), P is the north celestial pole, K is the North Pole of the ecliptic and Aries is the equinox




[1]

Let the Sun's right ascension and declination be ( ; ), Then $PS = 90^\circ -$   and spherical angle $KPS = 90^\circ +$  . Moreover, $PKS = 90^\circ -$  . Finally KS = 90°, since the Sun is on the ecliptic. The spherical triangle we must consider is clearly KPS. By the cosine formula

$$\cos(90^\circ - \ ) = \cos('') \cos(90^\circ) + \sin('') \sin(90^\circ) \cos(90^\circ)$$

i.e

$$\begin{aligned} \sin(\ ) &= \sin('') \sin(\ ) \\ &= \sin(23^\circ{:}5) \sin(40^\circ{:}4) = 0{:}2584 \end{aligned}$$

Hence,   = 15° [1]

Again by the four parts formula,

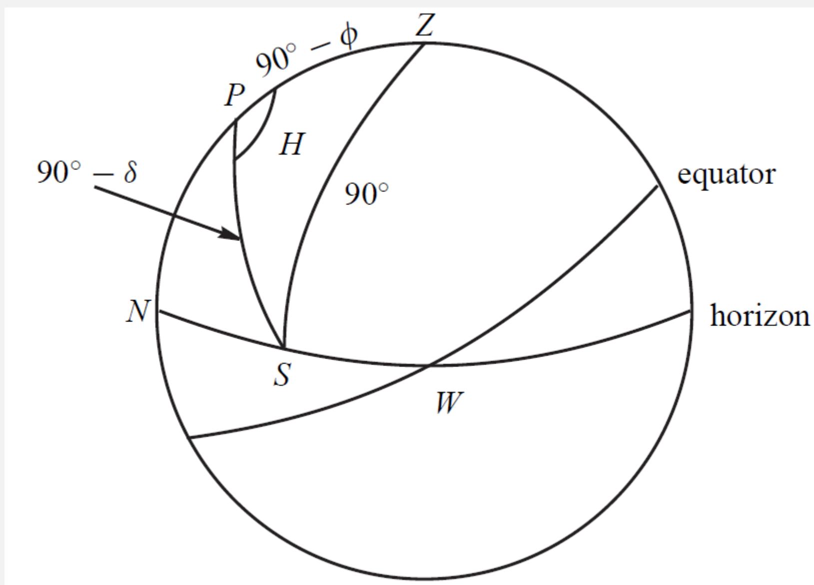
$$\cos \ \cos(90^\circ) = \sin \ \cot 90^\circ \sin(90^\circ) \cot(90^\circ + \ )$$

i.e

$$\cos\ \sin\ = \cos\ (\tan\ )$$
$$\tan\ = \cos\ \tan$$
$$= \cos 23^\circ\!:\!5 \tan 40^\circ\!:\!4 = 0\!:\!7805$$

Hence $\ = 38^\circ\!:\!0 = 2^h 32^m$ [1]

**c.** To calculate the hour angle of the Sun we need a new diagram. Let S be the position of the Sun at sunset, P the north celestial pole and Z the zenith point for the observer. Then $ZS = 90^\circ$. Further $PZ = 90^\circ -$ where is the latitude of the observer, namely 25°. PS is still $90^\circ -$ and the spherical angle ZPS is the required hour angle, H say.




[1]

Clearly we must consider the spherical triangle PZS. By the cosine formula

$$\cos 90^\circ = \cos(90^\circ - \ )\cos(90^\circ - \ ) + \sin(90^\circ - \ )\sin^\circ(90 - \ )\cos H$$
$$\cos H = -\tan\ \tan$$
$$= -\tan 25^\circ \tan 15^\circ = -0\!:\!1249$$

[1]

Hence the hour angle is found to be $97\!:\!18^\circ\!:\!4$ or $6^h 29^m$. The interval between sunrise and sunset is clearly 2H or $12^h 58^m$. Since the observer's longitude is 0 , both sunrise and sunset will occur at the same time for a point at the same latitude on the Greenwich meridian. So sunrise and sunset both are $6^h 29^m$ before and after noon (GMT). The times are, therefore, 5.31 am and 6.29 pm. [1]

# 3 Radius of the Earth

The scientist named Abu Reyhan Al-Biruni had successfully calculated the radius of the Earth in a different way with that done by Greek mathematician Erastosthenes. The calculation of the radius of the Earth by this method requires the top of a mountain with height $h$ and surrounded by a flat field. The angles of the slopes of the mountain with the ground, $_1$ and $_2$ at the points 1 and 2 are measured. The distance $d$ between point 1 and 2 also measured.

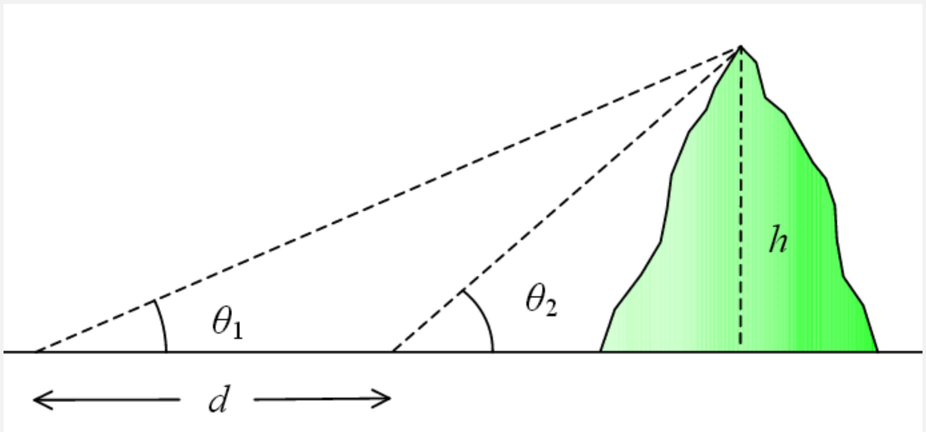
a. Express $h$ in terms of $d$; $_1$; and $_2$. [2]

An observer climbed the mountain peak and measured the angle ( $= 90° -$ ) between the vertical, and the tangent from the peak to the earth's surface. The point where the tangent touches the earth surface is the horizon. When the observer is on a higher mountain peak, the horizon will both farther and at a lower elevation.

b. Sketch the geometry of the mountain, the angle of decline of the horizon , and the radius of the Earth. [2]

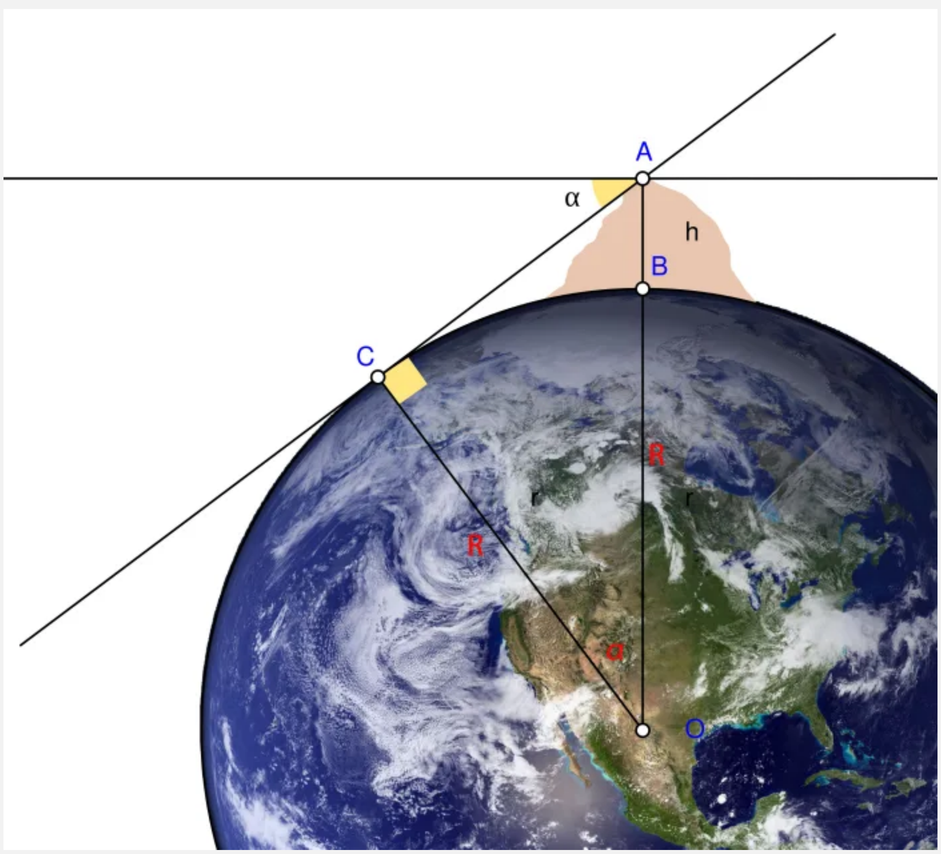
c. Explain the method of Al-Biruni in measuring the radius of the earth after calculate the height of the mountain $h$ and measure the angle of decline of the horizon . [2+2]

a. Judging from the diagram below,




$$h = \frac{d \tan \ _1 \tan \ _2}{\tan \ _2 - \tan \ _1}$$

b. The diagram below illustrates the situation:



c. From Law of sines,

$$\frac{AC}{\sin O} = \frac{R}{\sin A} = \frac{R+h}{\sin C}$$

$$\frac{AC}{\sin O} = \frac{R}{\sin(90^\circ - \quad)} = \frac{R+h}{\sin 90^\circ}$$

$$R = \frac{(R+h)\sin(90^\circ -}{sin 90^\circ}$$

So, $R = (R+h)\cos$

$$) \; R = \frac{h\cos}{1-\cos}$$

With his formula Biruni arrived at the value of the circumference of the earth within 200 miles of the actual value of 24,902 miles, that is less then 1% of error. Biruni's stated radius of 6335.725 km is also very close to the original value.

# 4 Kepler's Law

The person responsible for breaking us out of this very local volume and extending our reach to much larger distances was, well, two people, really. The first was Tycho Brahe, a Danish nobleman who took up astronomy in a big way. The telescope had not yet been invented, back in the late 1500s, but Tycho didn't let that stop him. He built a set of instruments which allowed him and his assistants to make very accurate measurements of stellar positions. In addition, he took meticulous notes of the data. After Tycho died soon thereafter in 1601, Kepler began to explore the rich treasure of measurements in Tycho's notebooks. He spent decades using the data to support a number of discoveries.

a. By conservation of energy, derive the formula for the escape velocity of an object. [2]

b. Consider a satellite in an elliptical orbit around a planet, with periapsis distance $a$ and apoapsis distance $b$. Derive a formula relating the total energy at $a$ to the total energy at $b$. [1.5+1.5]

c. Use Kepler's $2^{\text{nd}}$ law to find a formula relating $v_a$ and $v_b$ (Velocity of point a and b). You may approximate the areas as very thin triangles. [3]

d. Consider the satellite at periapsis and at apoapsis. Its velocity is directed perpendicular to the line connecting it to the planet. Will this satellite, if its velocity is equal to the escape velocity, be able to escape? [4]

e. What is the temperature of the satellite if the satellite is at $b = 4R_\oplus$ but farthest from Sun overall and the planet orbiting a sunlike planet from 2 AU distance. Consider the satellite has an area of $1\,m^2$ and albedo, $A_b = 0{:}7$. [4]

a. We know that,
Potential energy = -$\frac{GMm}{R}$
Kinetic Energy= $\frac{1}{2}mv^2$
Potential energy and kinetic energy at infinity $\Rightarrow R = \infty$ and $v = 0$
Potential energy and kinetic energy at $a \Rightarrow R = a$ and $v = v_{\text{escape}}$ Potential energy and kinetic energy at infinity = Potential energy and kinetic energy at $a$.

$$-\frac{GMm}{a} + \frac{1}{2}mv_{\text{escape}}^2 = 0$$

[1]

$$v_{\text{escape}} = \sqrt{\frac{2GM}{a}}$$

[1]

b. By conservation of energy, where $a$ is periapsis distance and $b$ apoapsis distance:

$$-\frac{GMm}{a} + \frac{1}{2}mv_a^2 = -\frac{GMm}{b} + \frac{1}{2}mv_b^2$$

c. Kepler's 2$^{\text{nd}}$ law:

$$\frac{av_a\Delta t}{2} = \frac{bv_b\Delta t}{2}$$

d.

$$-\frac{GMm}{a} + \frac{1}{2}mv_a^2 = -\frac{GMm}{b} + \frac{1}{2}m(\frac{a}{b}v_a)^2$$

$$\frac{1}{b} = \frac{GM}{a^2v_a^2} \pm \frac{GM}{a^2v_a^2} - \frac{1}{a}$$

Now, $b = a$;
Or,

$$b = \frac{a}{\left(\frac{v_{\text{escape}}}{v_a}\right)^2 - 1}$$

[1]

Observe that if $v_{\text{escape}} = v_a$, $b$ is infinite. Thus any object with the escape velocity must escape regardless of direction.

e. Given that, $b = 4R_{\oplus} = 4 \times 6{:}371 \times 10^6$ m. Here, the maximum distance over all will be, $d \approx 4R_{\oplus} \times 2\,AU$. [0.5]

For Equilibrium Temperature,

$$P_{\text{in}} = \frac{L_{\odot}}{4\ \ d^2} * (1 - A_b) * (\text{Area of Satellite receiving sunlight})$$

[1]

$$P_{\text{out}} = \ \ R_{\text{sat}}^2\ \ T_{\text{sat}}^4$$

[1]

If, $P_{\text{in}} = P_{\text{out}}$

$$\frac{L_{\odot}}{4\ \ d^2} * (1 - A_b) * (\text{Area of Satellite receiving sunlight}) = \ \ R_{\text{sat}}^2\ \ T_{\text{sat}}^4$$

$$T_{\text{sat}} = \sqrt[4]{\frac{L_{\odot}}{4\ \ d^2} * (1 - A_b) * 1\,m^2 * \frac{1}{R_{\text{sat}}^2}}$$

[1]

**0.5** Marks for right value.