# 5$^{\text{th}}$ Bangladesh Olympiad on Astronomy and Astrophysics

National Round 2022

May 21, 2022

**Instructions for the Candidate:**

- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Be sure to calculate the final answer in the appropriate units asked in the question.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.

Table 1: **Useful Constants and Formulas**

| | | | |
|---|---|---|---|
| Mass of the Sun | $M_\odot$ | $\approx$ | $1.989 \times 10^{30}\,\text{kg}$ |
| Mass of the Earth | $M_\oplus$ | $\approx$ | $5.972 \times 10^{24}\,\text{kg}$ |
| Mass of the Moon | $M_{☾}$ | $\approx$ | $7.347 \times 10^{22}\,\text{kg}$ |
| Radius of the Earth | $R_\oplus$ | $\approx$ | $6.371 \times 10^{6}\,\text{m}$ |
| Radius of the Sun | $R_\odot$ | $\approx$ | $6.955 \times 10^{8}\,\text{m}$ |
| Radius of the Moon | $R_{☾}$ | $=$ | $1737$ km |
| Distance from the Earth to the Moon | $r_{☾-\oplus}$ | $=$ | $384400$ km |
| Geometric albedo of the Earth | $A_{b,\oplus}$ | $\approx$ | $0.37$ |
| Geometric albedo Moon | $A_{b,☾}$ | $\approx$ | $0.12$ |
| Speed of light | $c$ | $\approx$ | $3 \times 10^{8}$ m/s |
| Synodic period of Moon rotation | | $\approx$ | 29.5 days |
| Astronomical Unit (AU) | $a_\oplus$ | $\approx$ | $1.496 \times 10^{11}\,\text{m}$ |
| Solar Luminosity | $L_\odot$ | $\approx$ | $3.826 \times 10^{26}\,\text{W}$ |
| Solar Constant | $S_\odot$ | $\approx$ | $1367\ \text{W/m}^2$ |
| Gravitational Constant | $G$ | $\approx$ | $6.674 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$ |
| 1 parsec | $1\,pc$ | $=$ | $3.086 \times 10^{16}\,\text{m}$ |
| Stefan's constant | $\sigma$ | $=$ | $5.670 \times 10^{-8}\,\text{Wm}^2\text{K}^{-4}$ |

# 1 Lunar Eclipse [12]

During a Lunar eclipse, the Moon traverses the Earth's shadow which consists of the penumbra and the umbra. Astrophotographers sometimes make a series of nice colorful pictures with the Moon disappearing and then reappearing. One of these series is shown in Figure 1, which covers the eclipse on January 21, 2019.

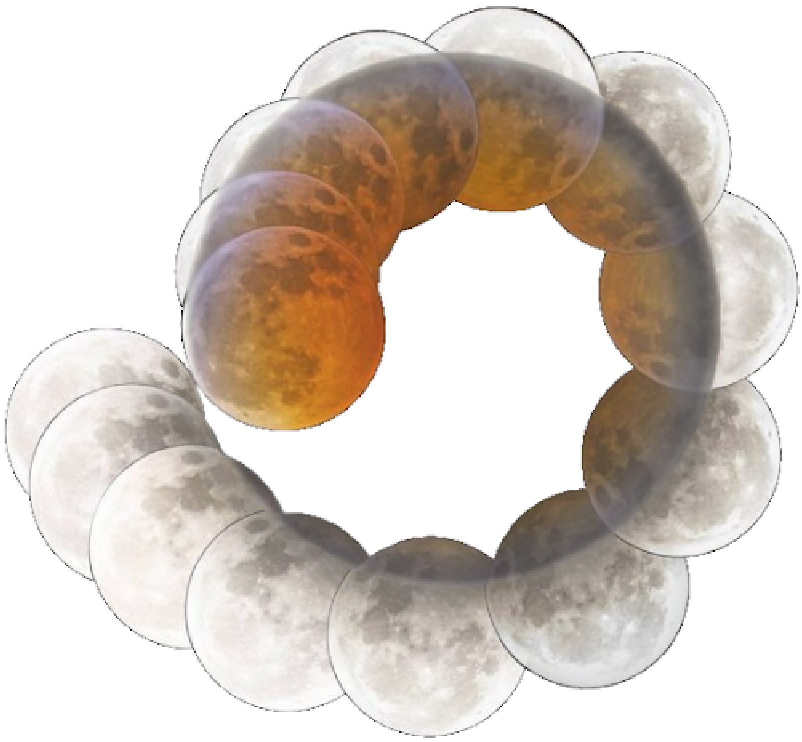


Figure 1. A series of pictures from the Lunar eclipse on January 21, 2019.

a. What type of lunar eclipse was this (total/partial/annular/penumbral etc.)? While passing through the penumbra, the Moon is dark grey. What color does the Moon have while passing through the umbra and why? [1]

b. What was the radius of Earth's shadow during this eclipse? Express your answer in km. [3] Assume that all celestial bodies are perfect spheres with equatorial radius.

c. Use Figure 1 to determine the Earth-Moon distance during this eclipse. [4]

d. Find out the range of angular size of the Moon for the earth day mentioned above so that the moon remains in a Total Eclipse. What would be this range for a Partial Eclipse? [4]

From your Astronomy class, you know that the eccentricity of the Moon's orbit is 0.0549.

## 2 The First Interstellar Comet [8]

The first interstellar comet was discovered on August 30, 2018, by the Crimean astronomer G.V Borisov, using a 65 cm telescope. Before entering the Solar System the comet was located in the celestial sphere near the star Ruchbah in the constellation Cassiopeia. On December 7, 2019 the comet passed through the perihelion of it's orbit, at a distance $r_p = 2.1$ AU from the Sun, at a speed of $v_p = 43$ km/s, from the Sun.

a. On that night, can we see the star Ruchbah ($\delta = 60°15'$) from Dhaka ($\phi = 24°$ N)? Why or why not? [2]

b. The parallax of the star is $p = 0.00328''$. Now estimate how long ago the comet passed near the star Ruchbah, assuming the radial velocity of the star Ruchbah is: [4]

1. $v_{\text{rad}} = 0$ km/s
2. $v_{\text{rad}} = -6.7$ km/s

c. Evaluate the possibility of seeing our Sun for an observer in Ruchbah with the naked eye. [2]

## 3 Globular Cluster [10]

The globular cluster NGC 6397 (Ara constellation) has an observed visual magnitude $m = 5.17$ mag and an angular diameter $\theta = 4.7'$. The Globular cluster has a parallax of $p = 0.42$ mas (1 mas $= 10^{-3}$ angular seconds). Assume that the stars in the cluster are all similar to the Sun on average.

a. Determine the distance to the globular cluster in km. [1]

b. Determine approximately how many stars N make up this cluster. [2]

c. Determine the approximate escape velocity from the cluster. [2]

d. What is the smallest diameter D (in cm) must a telescope have that would (theoretically) distinguish the individual stars in a cluster? Consider that we observe at a wavelength of $\lambda = 550$ nm. [2]

e. Calculate the stellar magnitude $\mu$ of the cluster area with an area of 1 arcsec$^2$. We are doing the observation from Chittagong, where the brightness of the sky reaches 20 mag $\cdot$ arcsec$^{-2}$? [3]

# 4 Earth Shine [10]

The Earth-shine is the dim glow of the unlit part of the Moon, which is illuminated by the sun's rays reflected from the Earth (upper moon in the following picture).

Phase angle, $\psi$ of an object is defined by the angle created by the earth and the sun at the object. Phase, $\phi$ is defined as the percentage of its disk that is illuminated by sunlight.

a. Prove that, $\phi = \cos^2 \frac{\psi}{2}$. You can use moon as an example. You may want to use diagram to illustrate your answer. [2]

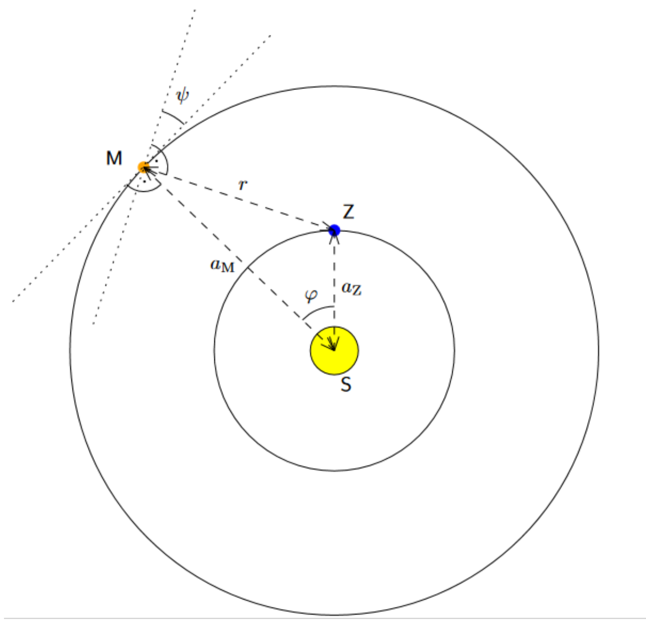
b. What is the relationship between the moon phase as seen from the earth, $\phi_{\leftmoon}$ and the earth phase as seen from the moon, $\phi_{\oplus}$ ? You can assume that $d_{\leftmoon - \odot} \approx d_{\oplus - \odot}$ [2]

c. We can think of moon illumination as the amount of reflected energy received from the moon in unit area of earth. For a given phase of moon, $\phi_{\leftmoon}$ , what will be the illumination caused by the earth-shine, as seen from the earth, $E_{\text{earth-shine}}$. Only use the constants provided in the front page and $\phi_{\leftmoon}$ . [4]

d. Find at what moon phase angle, $\psi_{\leftmoon}$ , the rate of change of earth-shine illumination will be highest? [2]

## 5 Retrograde Motion [23]

In this problem you will investigate the retrograde motion of mars. That is, a phenomenon when the apparent motion of the outer planets of our our solar system on the sky takes place (over a certain period of time) in the direction opposite to their true orbital motion around the sun. This is caused by the combined motion of both the planets and that of the earth. When both earth and the planet move around the Sun and then the planet which has a smaller orbit will tend to move faster and cause retrograde motion.




Let us denote by $T_E$ and $T_M$ the sidereal orbital period of the Earth and Mars, respectively. Let us also denote $a_E$ and $a_M$ the radii of the orbits pf the Earth and Mars, respectively. For the sake of simplicity let us assume that the orbits are circular.

The point of view we shall adopt is that of the heliocentric reference frame, which rotates with the same angular rate $\omega_E = 2\pi/T_E$ as the Earth orbits around the Sun. Let us denote this frame by $\mathsf{H_E}$.

a. If for any plane triangle, ABC with sides a, b, c and opposite angles A, B, C

$$\frac{b - a \ \cos C}{b \ \cos(C - a)} = \frac{\cos A}{\cos(C + A)}$$

Let's define $t_{\text{retro,planet}}$ as the time periods that a planet will remain at retrograde motion (in one synodic period) with respect to an observer of Earth.

Express $t_{\text{retro,planet}}$ in terms of the orbital radius of the planet, $a_{\text{planet}}$ and $a_E$. Using the equation determine the $t_{\text{retro,Jupiter}}$ and $t_{\text{retro,Saturn}}$ [3]

b. Find the angular frequency $\omega'_M$ of Mars' orbit around the Sun relative to the reference frame $\mathsf{H_E}$. State your answer in terms of $\omega_E$ and the sidereal angular frequency $\omega_M = 2\pi/T_M$ of Mars' orbit. [3]

Let us denote $\varphi$ the angle subtended between the radius vector (i.e. the line joining an object with the Sun) of the Earth and that of Mars.

c. Find the distance $r$ between Earth and Mars a function of $\varphi$. State your answer in terms of $a_M$, $a_E$ and $\cos\varphi$. [3]

Let us denote by $\psi$ the angle between the tangent to the Mars' orbit through the position of mars and the perpendicular to the line joining the Earth and Mars.

d. Find $\cos\psi$. State your answer in terms of $a_M$, $a_E$ and $\cos\varphi$. [3]

e. Find the transverse component $v_t$ of Mars' orbital speed relative to an Earth-based observer in the reference frame as a function of $\varphi$. State your answer in terms of $\omega_M$, $\omega_E$, $a_M, a_E$ and $\cos\varphi$. [3]

f. Find the angular speed $\omega''_M$ of Mars' motion on the Earth's sky relative to the background stars as a function of $\varphi$. State you answer in terms of $\omega_M$, $\omega_E$, $a_M, a_E$ and $\cos\varphi$. [3]

g. Find two numerical values $\varphi_{1,2}$ of $\varphi$ for which $\omega''_M$ vanishes. [2]

h. Find the duration $t_{\text{ret}}$ of Mars retrograde motion in one synodic period. [3]