6^{th} Bangladesh Olympiad on Astronomy and Astrophysics

National Round 2023

March 10, 2023

Instructions for the Candidate:

- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Be sure to calculate the final answer in the appropriate units asked in the question.
- Non-programmable scientific calculators are allowed.
- The mark distribution is shown in the [ ] at the right corner for every question.

Table 1: **Useful Constants and Formulas**

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	$R_☾$	$=$	1737 km
Distance from the Earth to the Moon	$r_{☾-\oplus}$	$=$	384400 km
Geometric albedo of the Earth	$A_{b,\oplus}$	$\approx$	0.37
Geometric albedo Moon	$A_{b,☾}$	$\approx$	0.12
Speed of light	c	$\approx$	3×10^{8} m/s
Synodic period of Moon rotation		$\approx$	29.5 days
Astronomical Unit (AU)	$a_\oplus$	$\approx$	1.496×10^{11} m
Solar Luminosity	$L_\odot$	$\approx$	3.826×10^{26} W
Solar Constant	$S_\odot$	$\approx$	1367 W/m^2
Gravitational Constant	G	$\approx$	6.674×10^{-11} Nm^{2}kg^{-2}
1 parsec	$1\,pc$	$=$	3.086×10^{16} m
Stefan's constant	σ	$=$	5.670×10^{-8} Wm^{2}K^{-4}

1 Phases of Moon and Mercury [5]

On a certain day, Oyon was observing the night sky. He was amused to see that the Moon is occulting the planet mercury which is a rare celestial occurrence.

a. Draw the geometry of the scenario. What is the maximum phase (the ratio of the area of the illuminated part of the disk to the total area) that the Moon can have in conjunction with Mercury? [2]

b. What will be the phase of Mercury? The orbits of Mercury and the Moon are considered circular and located in the plane of the ecliptic, the radius of Mercury's orbit is 0.4 AU. [3]

Solutions

Solution a

Obviously, the maximum phase of the Moon in conjunction with Mercury will be reached at the moments of maximum elongation of Mercury. Since we need the ratio of the area of the illuminated part to the total area of the disk, then the elongation can be both eastern and western, the areas of the illuminated part in both cases will be the same.

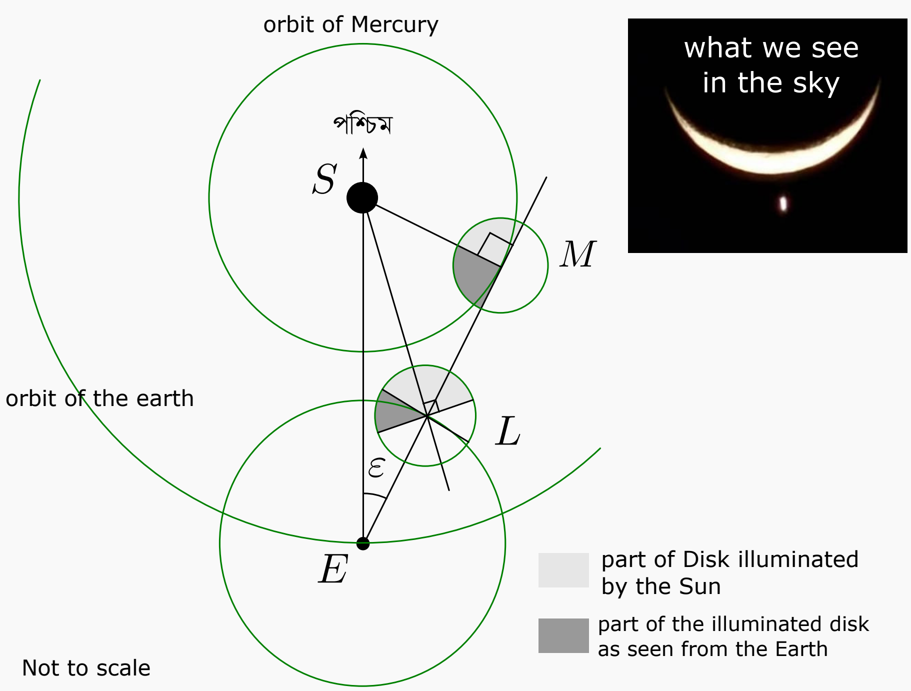


Figure 1: Conjunction of Mercury and Moon

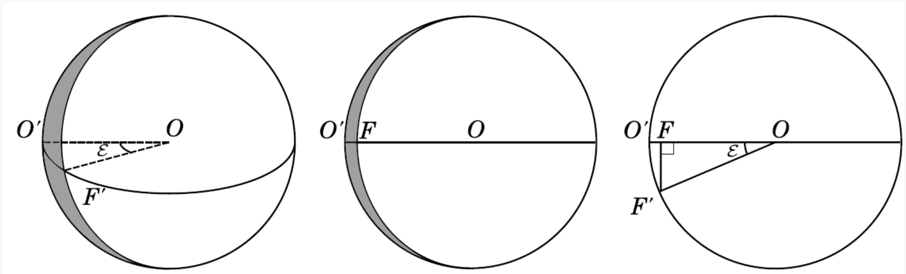
[Approximate drawing of the scenario carries **1 Mark**]

Let's estimate the maximum elongation of Mercury be ε (see Fig. 1). In the SPM triangle, the angle at M is straight, the hypotenuse NW is 1 AU, the leg of the SM is 0.4 AU. Hence, $\sin\varepsilon = 0.4$, i.e. $\varepsilon \approx 23°$. [Understanding the relation between phase and elongation **1 Mark**]

Solution b

It is obvious that in the maximum elongation the phase of Mercury is equal to 0.5. The moon has less phase.

Let's evaluate it. In the triangle ZSL, the side of the ZS is approximately 400 times larger than the side of the ZL, so we can assume that the angle at C is close to zero, and the angle at L is $180° - \varepsilon$. Therefore, the dihedral angle, "cutting out" a part of the area illuminated by the Sun and visible from the Earth, is equal to ε.



1b Cresent Projection of Moon

When projecting the image of the Moon onto the celestial sphere, a "crescent" will be obtained, shown in Fig. 1b, whose maximum width $O'F$ is defined as $OO' - OF$. $OO' = OF' = R_☾$, where $R_☾$ is the radius of the Moon.

$$O'F = OO' - OF = OO' - OF' \cos\varepsilon = R_☾ \,(1 - \sqrt{1 - \sin^2 \varepsilon}) \approx 0.1R_☾$$

[Understanding the crescent geometry carries **1 Mark**]

$$S = \frac{1}{2}\pi R_☾^2 - \frac{1}{2}\pi R_☾ \cdot 0.9R_☾ = 0.05\pi R_☾^2$$

and the ratio of the area of the illuminated part of the disk to the total area

$$\frac{S_{moon}}{S_{disk}} = \frac{0.05\pi R_☾^2}{\pi R_☾^2} = 0.05$$

From the phase, it is evident that the moon is near New at this particular time due west after sunset. [Final ratio carries **1 Mark**]

2 Dimensions of Blackhole [15]

We will venture into the mysterious realm of quantum gravity, and use dimensional analysis to learn about black holes. Here, all of the fundamental constants will be involved. First, we can define it a little more formally, "*A black hole is a region of spacetime that traps light. The boundary of this region is called the event horizon*".

a. Express the radius of a black hole, i.e. light-trapping region, in terms of its mass, m, gravitational constant, G and the speed of light, c using only dimensional analysis. [2]

b. Using the equation you derived, find the radius of a black hole with a mass of 2×10^{31} kg. [1]

c. The actual size of a black hole with a mass of 2×10^{31} kg is nearly 29.7 km. If this is different from the answer you found in (b), suggest a reason why. [1]

In a section of Black Holes Ain't So Black (Chapter 7, pp. 109-110), Hawking writes the following:
"*(. . .) a black hole ought to emit particles and radiation as if it were a hotbody with a temperature that depends only on the black hole's mass (. . .)*"
At this point, Hawking goes on to explain in detail the quantum properties of black holes, as well as the concept of Hawking temperature. The British physicist's explanations end with the following words:
"*(...) A flow of negative energy into the black hole therefore reduces its mass (. . .) Moreover, the lower the mass of the black hole, the higher its temperature.*"

d. Guess the Temperature of black holes using the same approach. If you're unable to solve a. then use Schwarzschild radius as a proxy. [3]

The deflection of light by a gravitational field was first predicted by Einstein in 1912, a few years before the publication of General Relativity in 1916. A massive object like Blackhole that causes a light deflection behaves like a classical lens. This prediction was confirmed by Sir Arthur Stanley Eddington in 1919.

We'll now use dimensional analysis to determine the form of the equation describing the deflection angle due to gravity for a light ray passing by a star (or other objects) of mass m. First, let's define the angle θ as the angle between the directions of the ray of light when it is asymptotically far from the star (coming towards the star and going away from the star), as shown in

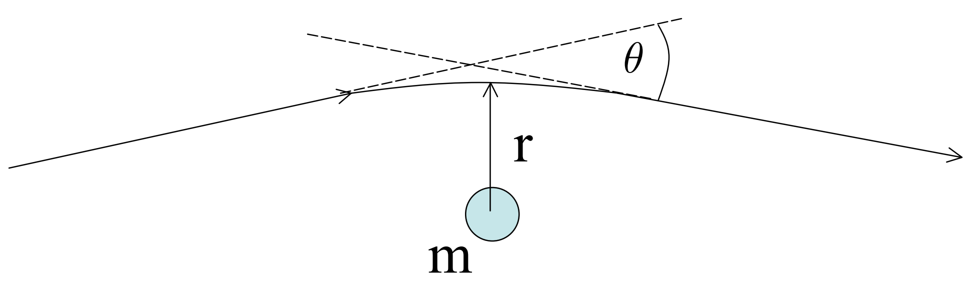


Figure 2: Deflection of light by a Blackhole

Note that when angles appear in an equation, they should always be expressed in radians. An angle expressed in radians is dimensionless. Therefore, the deflection angle θ is dimensionless.

e. On which physical variables might the deflection angle depend? [4]

f. Using general relativity the proportionality constant from the previous answer would be $k = 4$. If the central object is the Sun calculate the numerical value of the deflection angle of a photon. [4]

Solutions

Solution a

We suppose that the size R of blackhole is connected with mass m,

$$R \sim m^a\, G^b\, c^d$$

[Understanding the relation between parameters carries no mark]

Taking dimensions L remains in leftside and rightside

$$M^a \cdot \left(\frac{L^3}{MT^2}\right)^b \cdot \left(\frac{L}{T}\right)^d = M^{a-b} L^{3d+b} T^{-2b+d}.$$

Equating exponents in both sides [Solving equations carries **1 Mark**]

$$a - b = 0, \quad 3b + d = 1, \quad 2b + d = 0$$

Solving the above equations we get $a = b = 1$ and $d = -2$ from which we understand that,

$$R \sim \frac{Gm}{c^2} \Longrightarrow R_s = \frac{2Gm}{c^2}$$

[Final equation carries **1 Mark**]

The student is meant to ignore the factor of '2' to get full marks.

Solution b

We derived in (a),

$$R_s = \frac{Gm}{c^2}$$

$$R_s = \frac{G \times 2 \times 10^{31}}{c^2}$$

[Plugging m into the right equation carries no mark]

$$R_s = 14830\ m \sim 14.8\ \text{km}$$

[Final calculations **1 Mark**]

Solution c

Dimensional analysis doesn't consider the effect of dimensionless quantities such as numbers into the equation. [Understanding of this idea **1 Mark**]

Solution d

We can guess that

$$\mathcal{T} \sim R^a\, k_B^b\, \hbar^d\, c^e$$

[Understanding the relation between parameters carries **1 Mark**]

The dimensions of leftside is Temperature and the dimensions of rightside is

$$L^a \cdot \left(\frac{ML^2}{\Theta T^2}\right)^b \cdot \left(\frac{ML^2}{T}\right)^d \cdot \left(\frac{L}{T}\right)^e = L^{a+2b+2d+e} M^{b+d} T^{-(2b+d+e)} \Theta^{-b}$$

We got a systems of equations [Solving equations carries **1 Mark**]

$$\begin{aligned} a + 2b + 2d + e &= 0 \\ b + d &= 0 \\ 2b + d + e &= 0 \\ -b &= 1 \end{aligned}$$

Solving the equations we get, $a = b = -1$ and $d = e = 1$, Therefore the temperature of blackhole can be approximated as

$$\mathcal{T} \sim \frac{\hbar c}{k_B R} \sim \frac{\hbar c^3}{k_B G m}$$

[Final equation carries **1 Mark**]

Solution e

Our physical intuition tells us that the angle should depend on the mass of the star m and on the distance of the ray of light from the star. Let's define r to be the distance of the closest approach of the ray to the star as shown in the sketch above. If we proceed with our dimensional analysis at this point, we will find that there is no dimensionally consistent form for the equation expressing θ in terms of m and r, just as we found that the period of oscillation of a pendulum could not be expressed in terms of m and l alone. So, again there must be a dimensional constant that we need to include. Since the deflection of light is due to gravity, we might suspect that the angle depends on the gravitational constant G. What are the dimensions of G? Recall that the equation for the gravitational force between two massive objects of mass m_1 and m_2 a distance r apart is given by $F = \frac{Gm_1 m_2}{r^2}$. Therefore,

$$[G] = \left[\frac{Fr^2}{m_1 m_2}\right] = \mathrm{M^{-1}L^3T^{-2}}$$

where we used $[F] = \mathrm{MLT^{-2}}$. Now let's try to find the equation for θ:

$$\theta = k m^\alpha r^\beta G^\gamma$$

The equation relating dimensions is

$$\mathrm{M^0L^0T^0} = \mathrm{M^\alpha L^\beta (M^{-1}L^3T^{-2})^\gamma}$$

Equating the exponents of the basic dimensions M, L, and T, we get

$$\text{Exponents of } M \to 0 = \alpha - \gamma,$$

$$\text{Exponents of } L \to 0 = \beta + 3\gamma,$$

$$\text{Exponents of } T \to 0 = -2\gamma.$$

But the last equation gives us $\gamma = 0$, the second one gives us $\beta = 0$ and the first one gives $\alpha = 0$! So, we must still be missing a physical variable or a dimensional constant. Which dimensional constant is most likely to be relevant for the case of the bending of light by gravity? How about the speed of light, c? Let's try it:

$$\theta = km^\alpha r^\beta G^\gamma c^\delta$$

[Understanding the relation between parameters carries **2 Mark**]

The equation relating dimensions is now

$$\mathrm{M^0L^0T^0} = \mathrm{M^\alpha L^\beta (M^{-1}L^3T^{-2})^\gamma (LT^{-1})^\delta}$$

Equating the exponents of the basic dimensions M, L and T, we get

$$\text{Exponents of } M \to 0 = \alpha - \gamma,$$

$$\text{Exponents of } L \to 0 = \beta + 3\gamma + \delta,$$

$$\text{Exponents of } T \to 0 = -2\gamma - \delta.$$

So now we have three equations in four unknowns. The four exponents α, β, γ, and δ are constrained but are not uniquely determined. Each of the three equations involves γ, so let's express the other three exponents in terms of γ. From the first equation, $\alpha = \gamma$. From the last equation, $\delta = -2\gamma$. And from the second equation, $\beta = -\delta - 3\gamma = 2\gamma - 3\gamma = -\gamma$. [Solving equations carries **1 Mark**]

Therefore, the equation for the bending angle is of the form

$$\theta = km^\gamma r^{-\gamma} G^\gamma c^{-2\gamma} = k\left(\frac{mG}{rc^2}\right)^\gamma$$

Actually, there could be more than one term in the equation for θ, each with a different value of the exponent γ and the constant k, but each term must have the above form. In fact, there could be an infinite number of terms (an infinite series), in which case the right-hand side might be a function of $\frac{mG}{rC^2}$ that can be represented as a series expansion. So, we have not uniquely determined the form of the equation for θ but we can already draw some conclusions from the above equation. For example, we can see that the bending angle depends on the ratio m/r; if m and r are both changed by the same factor, the bending angle will be the same.

We can go further and restrict γ by using physical intuition. First, we expect that θ approaches zero as m becomes very small or as r becomes very large. If γ were negative, then θ would approach infinity as m became small or r became large. Therefore, γ must be a positive exponent: $\gamma > 0$.

To further restrict γ, we can try to apply physical intuition to the derivative of θ with respect to m or r. Since the ratio m/r appears in the equation for θ, let's consider the derivative with respect to $x = \frac{mG}{rc^2}$:

$$\frac{d\theta}{dx} = \gamma k x^{\gamma - 1}, \quad \gamma > 0$$

Our physical intuition might tell us that in the limit of $x = \frac{mG}{rc^2}$ becoming very small, the change in θ with respect to a change in m/r should become small, but should not vanish. Therefore, we want the exponent of $\frac{mG}{rc^2}$ to be zero in the equation for $\frac{d\theta}{dx}$. So γ must equal 1 and the equation for θ, at least for small values of the dimensionless combination of variables $\frac{mG}{rc^2}$, is

$$\theta = k\frac{mG}{rc^2}$$

[Final equation carries **1 Mark**]

I admit that this last argument is a bit of a stretch...

What about the dimensionless constant k? A survey of all the equations that you will learn in the Introductory Physics sequence will convince you that the dimensionless constants in physical equations are always of order 1. And the equation we just derived is no exception. It turns out that $\theta = k\frac{mG}{rc^2}$ with $k = 4$.

3 Rotation Rate of a Pulsar [20]

A Pulsar is a Neutron-Star with a very high rotation rate. But how fast can a neutron-star rotate before it breaks up due to its own centrifugal forces? We assume the pulsars to be spheres of uniform density, which are gravitationally bound.

a. Find an expression for the minimum rotation period, P_{min}, of a pulsar as a function of its mass, M, and radius, R, before it breaks up. [2]

b. Evaluate P_{min} for a typical pulsar with $M = 1.4M_{\odot}$ and $R = 10\ km$. [1]

c. Millisecond pulsars are those that have rotation periods on the order of a millisecond. The fastest-rotating millisecond pulsar rotates about 716 times per second. What limit does this put on its density? [3]

A pulsar is formed from a massive progenitor star which typically has a magnetic field of $0.1T$ and an average density of $0.1\ kg/m^3$. Such a star loses about 90% of its mass toward the end of its life. The remnant mass forms a pulsar of the kind described above.

d. Assuming that the magnetic flux is conserved during the formation of a pulsar, find the typical magnetic field at the surface of the pulsar. For the pulsar $M = 1.4M_{\odot}$ and $R = 10km$. [4]

The Crab Nebula has a total luminosity of $\sim 5 \times 10^{31}$ J/sec. This nebula is powered by a centrally located pulsar which is now rotating with a period of 0.033 seconds. The pulsar is observed to be slowing down (the period is increasing), and the corresponding decrease in its supply of stored rotational kinetic energy is used to inject power into the Crab Nebula. Assume a mass and radius for the pulsar of $1.4M_{\odot}$ and $10km$, respectively.

e. Write down an expression for the rotational kinetic energy ($I\omega^2/2$) in terms of $M,\ R$ & P of the pulsar. [Approximate the moment of inertia, I, as that for a uniform density sphere, i.e., $2MR^2/5$.] [2]

f. Compute how much energy is stored in the Crab pulsar in the form of rotational kinetic energy. [1]

We define E as the rotational kinetic energy, $\dot{E}$ as the rate of change of this energy with time, and $\dot{P}$ as the rate of change of the Pulsar's period with time. You may note that if $E \propto P^n$ then $\dot{E}/E = n\dot{P}/P$

g. Using your answer in part (e), find a general expression for the rate at which rotational kinetic energy is lost as the period of pulsar increases (i.e., as the rotation rate of the pulsar decreases). Express your answer in terms of $R,\ M,\ P$, and $\dot{P}$. [2]

h. Evaluate this expression for the Crab pulsar, given the fact that the observed spin-down rate $\dot{P}$. amounts to 36 nanoseconds per day. Compare this to the power output of the entire Crab Nebula. [2]

i. Use the spin-down rate given in part (h) to make a rough estimate of the age of the Crab pulsar and hence that of the nebula. Compute the actual age from the fact that the supernova that led to the formation of the pulsar was observed by the Chinese in the year 1054 A.D. [3]

Solutions

Solution a

$$\omega^2 R = GM/R^2$$

[Equating centrifugal to gravitational acceleration **1 Mark**]

$$\omega = \sqrt{GM/R^3}$$

$$P_{min} = \frac{2\pi}{\sqrt{GM/R^3}}$$

[Finding the final expression **1 Mark**]

Solution b

$$P_{min} = \frac{2\pi}{\sqrt{G \times 1.4 \times 2 \times 10^{30}/(10 \times 10^3)^3}}$$

$$P_{min} = 4.596 \times 10^{-4}\ r\text{sec}$$

[Calculating the final value **1 Mark**]

Solution c

Rotation rate, $f = 716Hz$
Period, $P_{min} = 1/716$ sec
$= 1.400 \times 10^{-3}$ sec [**1 Mark**]

from (a) $P_{min} = \dfrac{2\pi}{\sqrt{GM/R^3}}$ and putting $M = \frac{4}{3}\pi R^3 \rho_p$

$$\rho_p = \frac{3\pi}{GP_{min}^2} \qquad [\textbf{1 Mark}]$$

$$\rho_p = 7.21 \times 10^{16} kg/m^3$$

[Calculating the final value **1 Mark**]

Solution d

$$R_{st} = \sqrt[3]{\frac{3M_{st}}{4\pi\rho_{st}}} = \sqrt[3]{\frac{30M_p}{4\pi\rho_{st}}} \qquad [\textbf{1 Mark}]$$

$$R_{st} = 4.15 \times 10^{10} m$$

$$\phi = AB = constant$$

[**1 Mark**]

$$4\pi R^2 B = constant$$

$$B_{st}/B_p = R_{st}^2/R_p^2$$

[**1 Mark**]

$$B_p = B_{st} \times R_p^2/R_{st}^2$$

$$B_p = 3.4 \times 10^{11} \text{ T}$$

[Calculating the final value **1 Mark**]

Solution e

$$\frac{I\omega^2}{2}$$
$$\frac{1}{2} \times \frac{2MR^2}{5} \times (\frac{2\pi}{P})^2$$
$$\frac{4\pi^2 MR^2}{5P^2}$$

[**2 Mark**]

Solution f

Rotational energy $= 4\pi^2 \times 1.4 \times 2 \times 10^{30} \times (10 \times 10^3)^2/5(0.033)^2$ J
$= 2.03 \times 10^{42}$ J [Calculating the final value **1 Mark**]

Solution g

Here, $E \propto P^{-2}$
So, $\dot{E}/E = -2\dot{P}/P$ [**1 Mark**]
From (e),

$$\dot{E} \times \frac{5P^2}{4\pi^2 MR^2} = -2\dot{P}/P$$

$$\dot{E} = -\frac{8\pi^2 MR^2}{5P^3}\dot{P}$$

[**1 Mark**]

Solution h

Convert 36 ns/day to sec/sec, [**1 Mark**]
Plug in the numbers, and calculate to get 5.1e31 J/s which is the same as the luminosity [**1 Mark**].

Solution i

$P/\dot{P} = 2511$ yrs [**3 Mark**]

4 Measuring the Astronomical Unit with the Venus Transit [8]

After Kepler published his third law in 1619, the distances between the planets in the Solar System were all known relative to the distance between the Earth and the Sun (1 Astronomical Unit). A measurement of one of these distances was therefore enough to determine the physical length of 1 Astronomical Unit (AU). One of the first measurements was conducted making use of the Venus transit in 1761, as proposed by British astronomer Edmund Halley. The aim of this problem is to follow the main steps in his derivation and to obtain our own estimate of the AU.

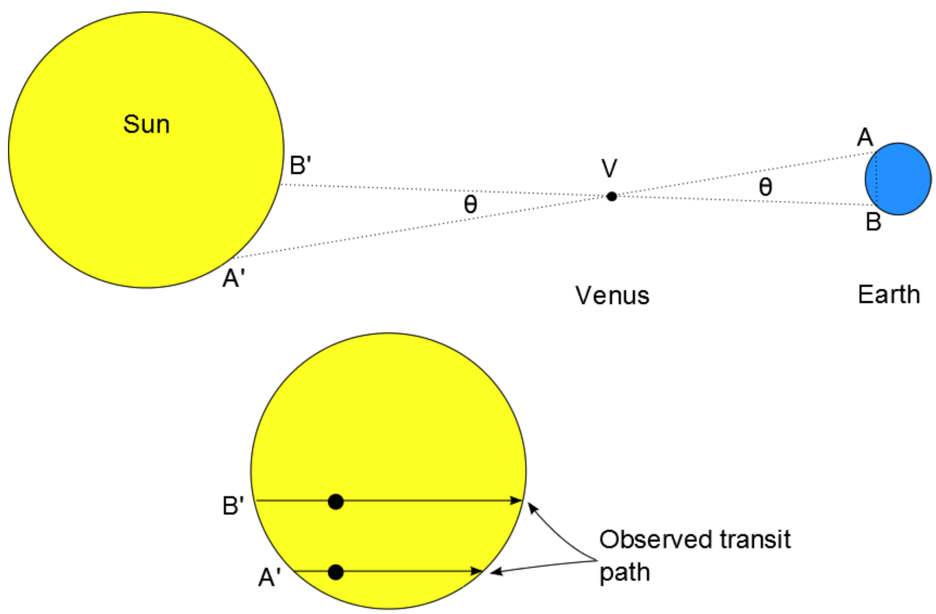


Figure 3: Geometry of Sun, Earth, and Venus system

As shown in Fig-3 the transit of Venus will be observed at slightly different positions when viewed from different points on Earth. Knowing the parallax of Venus θ_V and the distance between observers A and B the Earth-Venus distance can be calculated, and therefore all distances between the planets in the Solar System. Halley proposed to infer θ_V by timing the duration of the Venus transit at two different locations on Earth A and B.
In order to estimate the AU, we will use data from 2004's Venus transit observed in Cairo (A) and Durban (B).

To solve this problem, we will make the following assumptions:

1. The Earth does not rotate.

2. The Venus transit happens when Venus is below the Earth-Sun plane.

Location	Start of transit	End of transit
Cairo (A)	5:39:09	11:04:35
Durban (B)	5:35:52	11:10:07

Table 2

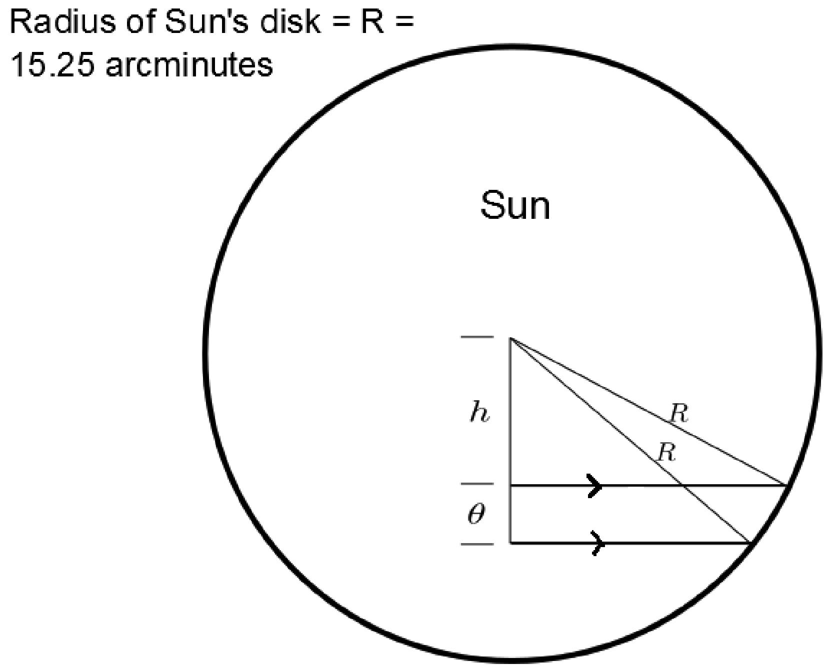


Figure 4: Geometry of Venus transit.

a. How long do the transits last in Cairo, T_A and in Durban, T_B? [1]

b. Assuming small angles, determine the angular separation between the two Venus transits θ using Fig-4 and Table 2. The angular size of Sun's disk is given by $R = 15.25$ arcmin and the angular velocity of Venus is $d_V = 0.0669$ arcsec/sec. [2]

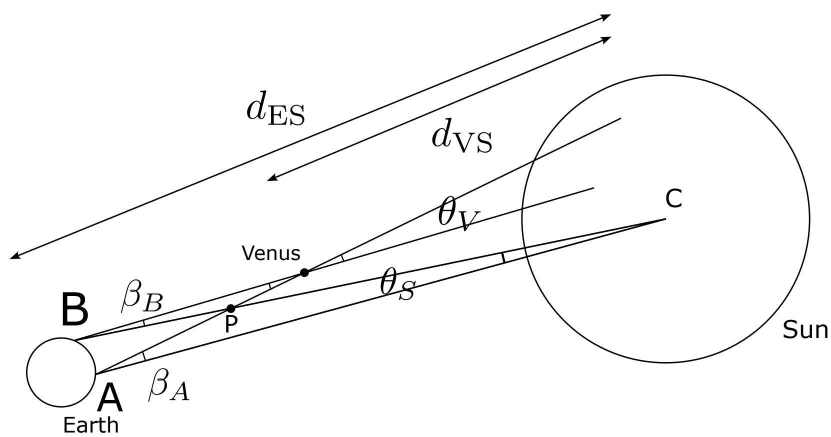


Figure 5: Effect of Solar parallax on measurement of Venus transit

c. Using the angle θ calculated in the previous problem, determine the parallax of Venus θ_V as a function of the parallax of the Sun θ_S using Fig 5. [2]
Hint: Relate the angle θ to the two angles β_A and β_B.

d. Assuming that the distance between the two positions A and B is $d_{AB} = 5840$ km and using Kepler's third law

$$\frac{T^2}{a^3} = \text{const.}$$

where a is the semi-major axis of the planetary orbit and T is its orbital period, determine the Astronomical Unit using $T_{Venus} = 224$ days and $T_{Earth} = 365$ days. You can work with the assumptions that planets move on circular orbits around the Sun. [3]

Solutions

Solution a

For Cairo, T_A = Ending time - Starting time $= (11 \times 3600 + 4 \times 60 + 35) - (5 \times 3600 + 39 \times 60 + 09)$ $= 19526\ s$
Similarly for Durban, $T_B = 20055\ s$ [For both calculations **1 Mark**]

Solution b

If we assume that the angular separations in the problem are small, we can work in a planar geometry and apply Pythagoras' theorem to get the angular separation of the two transits. From Fig-1 we have that

$$\theta = \sqrt{R^2 - \left(\frac{T_A D_V}{2}\right)^2} - \sqrt{R^2 - \left(\frac{T_B d_V}{2}\right)^2}$$

[Using pythagoras to calculate θ, **1 Mark**]

Inserting

$$\begin{aligned} T_A &= 11:04:35 - 5:39:09 = 19526 \text{ s} \\ T_B &= 11:10:07 - 5:35:52 = 20055 \text{ s} \\ d_V &= 0.0669 \text{ arcsec/sec} \end{aligned}$$

We get,

$$\theta \approx 18.5 \text{ arcsec}$$

[Using the values provided to find θ, **1 Mark**]

Solution c

Since each observer measures the position of Venus relative to "his own Sun" we need to account for the fact that they see the Sun under slightly different angles. This is shown in Fig 5. Looking at the triangles ΔAPC and ΔBP-Venus, we have [Understanding the relation between triangle ΔAPC and ΔBP-Venus **1 Mark**]

$$\beta_B + \theta_V = \beta_A + \theta_S$$

and therefore $\theta_V = \beta_A - \beta_B + \theta_S$. The quantity $\beta_A - \beta_B$ is the angular separation θ calculated in (1).

Solution d

To determine the parallax of the Sun, we need to know the distance from Earth to Venus in terms of the AU. Kepler's third law allows us to relate the distances from Venus and Earth to the Sun to their orbital periods, so if $d_{ES} = 1$ AU then

$$d_{VS} = \left(\frac{T_V}{T_E}\right)^{\frac{2}{3}} d_{VS} \approx 0.72 \text{ AU}$$

Therefore $d_{ES} = 0.28$ AU. Using $d_{Ei} = \dfrac{d_{AB}}{2\tan\dfrac{\theta_i}{2}}$, where $i = V$, S and the small angle approximation $\tan\frac{\theta_i}{2} \approx \frac{\theta_i}{2}$ we can therefore relate the parallax of Venus and the Sun through

$$\theta_S = \frac{d_{EV}}{d_{ES}}\theta_V$$

which allows us to calculate θ_S as $\theta_S = \left(\dfrac{d_{ES}}{d_{EV}} - 1\right)^{-1}(\beta_A - \beta_B) = 7.19$ arcsec.
The distance Eath-Sun can be calculated with

$$d_{ES} = \frac{d_{AB}}{2\tan\dfrac{\theta_S}{2}}$$

to be

$$1 \text{ AU} \approx 1.66 \times 10^8 \text{ km}$$

[Final answer carries **1 Mark**, 20% deviation of provided answer will receive **0.5 Mark**]

5 Robinson Crusoe: Lost in the sands [8]

The observation night

Deserts are one of the places with least sky pollution, which makes it the best spot for night sky observations. So, you and some of your friends decided to go on a tour to a desert. As you are the only astronomer in the team, you have to help your friends to point the telescope to the right direction.

Here is the map of the sky of the observation night.

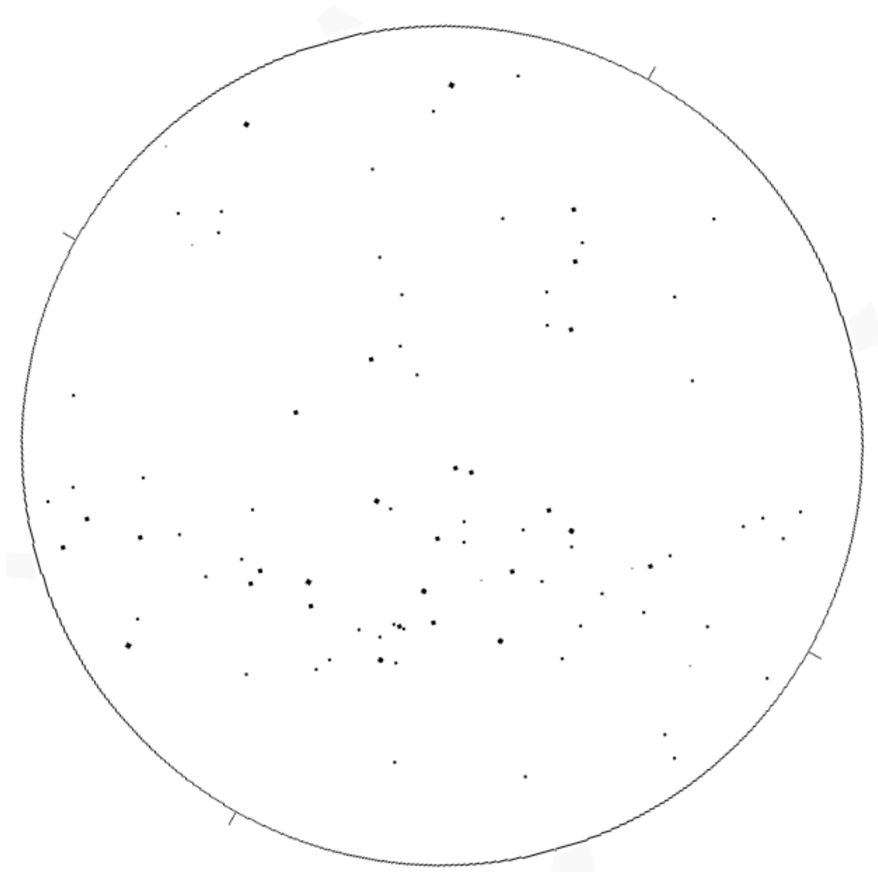


Figure 6: Sky of Observation Night

a. Mark the cardinal points on the map as N, S, E, and W. [0.5]

b. On the map, draw at least 4 constellations you know so that it becomes easier for you to find the Deep Sky Objects. Label them as C1, C2, C3, C4, and write the names in the table. [2]

The real adventure

You love roaming around in the night alone. Lost in the beauty of the night sky, you started walking alone after all of your friends are asleep. However, suddenly you realized that you came too far away from your camp, and can't find the way back. After three exhausting days of searching for the way back to your friends, you finally came to the camp. But alas! Your friends probably left after searching for you, and finally thinking that you're dead.
Here starts your adventure. You have some dry foods and water in the tents with which you can survive for

a few days. But you need to find your way back to home.

Fortunately, you know the location of the station from where you came, and unfortunately, you don't know your current location. However, you have a weird clock that shows the local sidereal time of Greenwich – a birthday gift from your best friend that you always keep with yourself. Armored with the knowledge of star maps and positional astronomy, you have to find your way back to home. You can use the following map.

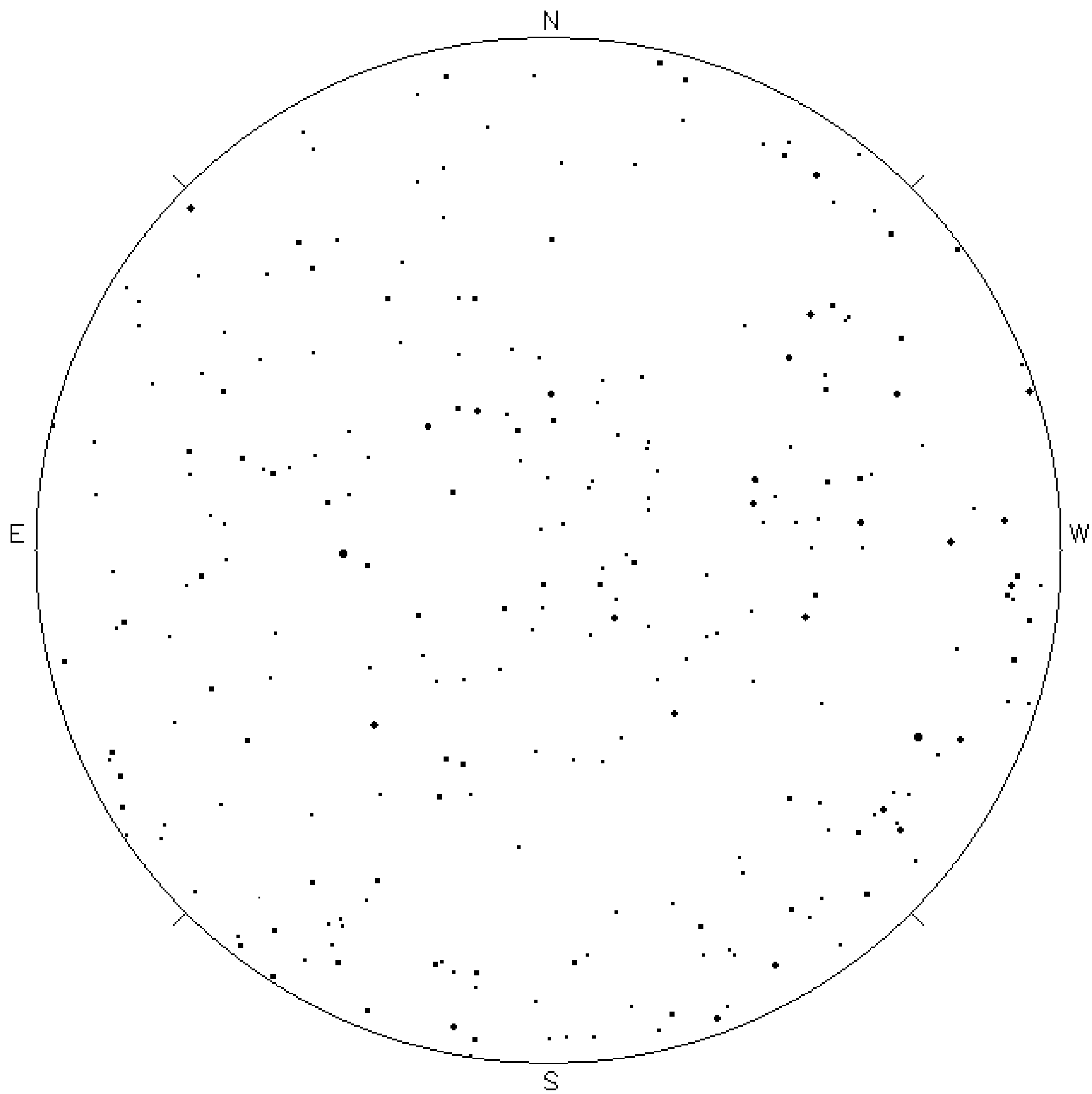


Figure 7: Sky map

a. Estimate your latitude from the star map. [0.5]

b. From the sky map, calculate your Local Sidereal Time. [1.5]
Hint: What is the actual definition of Local Sidereal Time?

c. Your watch shows the current Greenwich Sidereal Time $00^h\,18^m\,33.6^{ss}$. Calculate the longitude of your location. [1.5]

d. Given the location to the nearest human habitation to be 26.5°N, 27.6°E, calculate how far you're away from there (In nautical miles). [1]

e. Determine the angle to the direction to which you should start walking in order to go back to the human habitation. [1]

Rough Answers

Your location: $28° N, 23.36° E$
HA of Autumnal Equinox = 332°
LST = $360° - 332° = 28° = 1^h\,52^m$
GST = LST + $\lambda_{west} = 28° - 23.36° = 4.64° = 18^m\,33.6^{ss}$

Solutions

Solution a

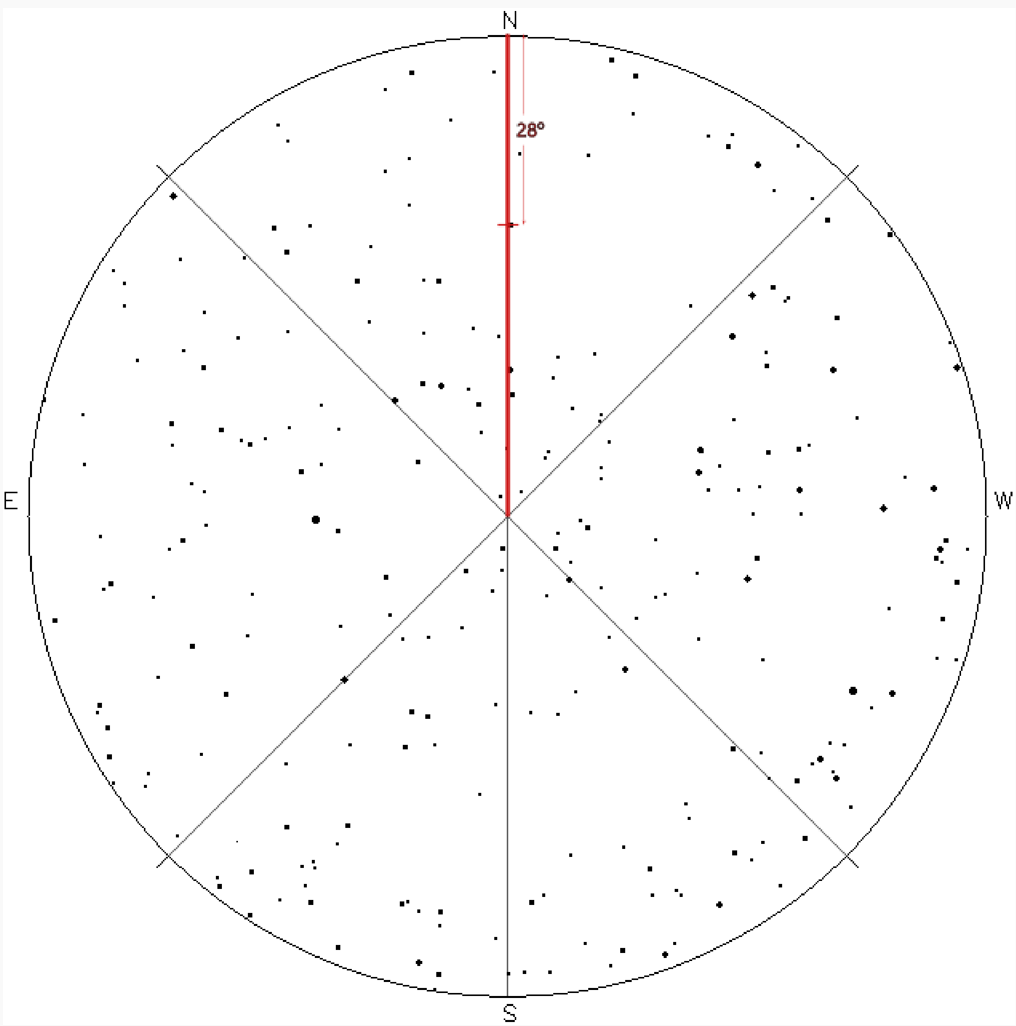


Figure 8: Latitude

Latitude = 28° N [0.5 pt for estimating the latitude from the map.]

Solution b

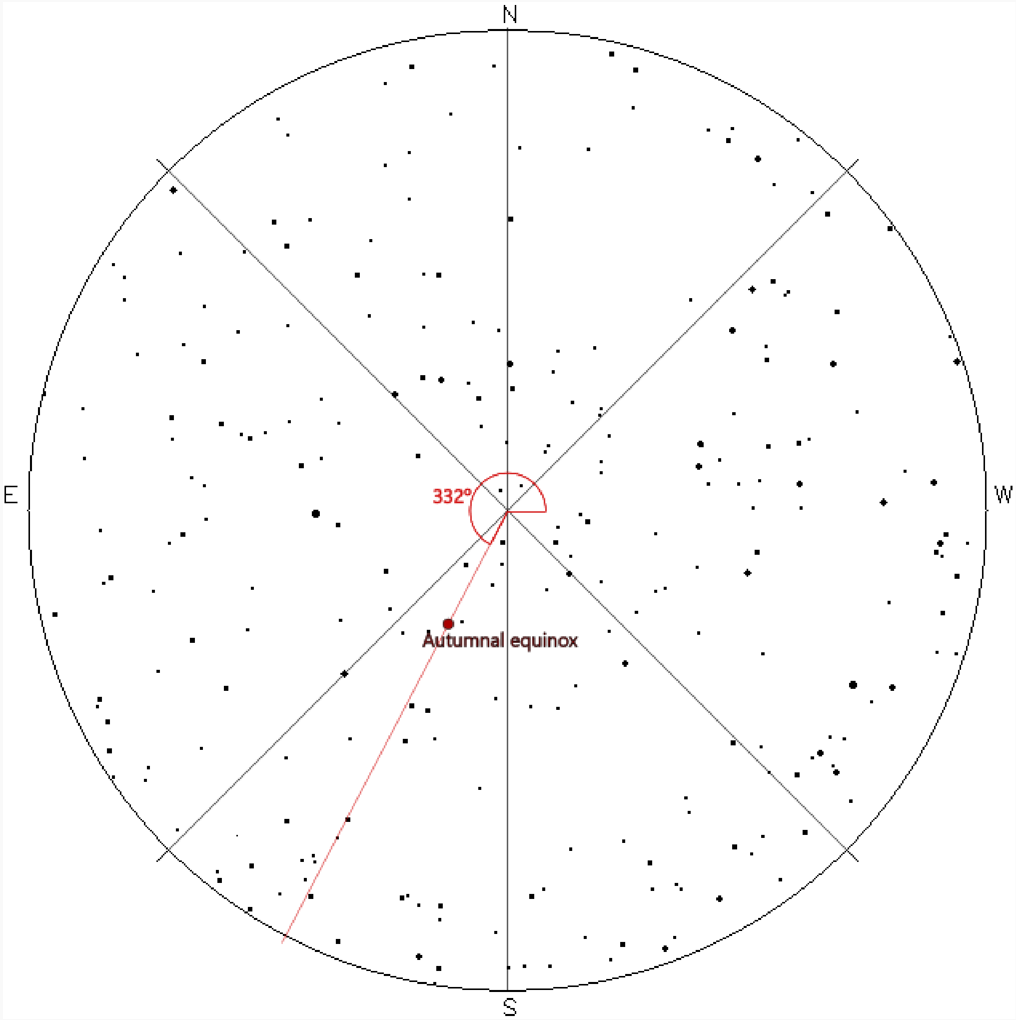


Figure 9: LST

Local Sidereal Time is the Hour Angle of Vernal Equinox. In the map we can find the Autumnal Equinox at 332°. Hence, the HA of Vernal Equinox is [1 pt for understanding that LST is HA of Vernal Equinox]

$$360^\circ - 332^\circ = 28^\circ = 1^h\,52^m$$

$$\therefore \text{LST} = 1^h\,52^m$$

[0.5 pt for numerical value]

Solution c

$$\lambda_{west} = \text{GST} - \text{LST}$$

[1 pt for formula]

$$\lambda_{west} = 18^m\,33.6^{ss} \times 15^\circ - 1^h\,52^m \times 15^\circ = -23^\circ\,21'\,36"$$

$$\therefore \boxed{\lambda = 23^\circ\,21'\,36" \text{ E}}$$

[0.5 pt for numerical value]

Solution d

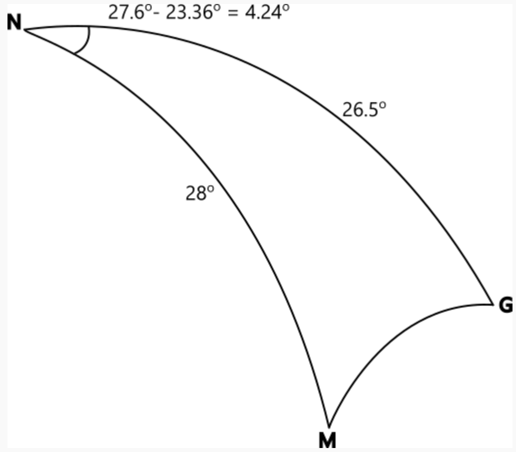


Figure 10: Cosine rule

$$\begin{aligned}\cos MG &= \cos NM \cos NG + \sin NM \sin NG \cos \angle MNG \\ &= \cos 28^\circ \cos 26.5^\circ + \sin 28^\circ \sin 26.5^\circ \cos 4.24^\circ \\ \therefore MG &= 2^\circ\ 27'\ 9.16'' \approx 2.4525^\circ\end{aligned}$$ [0.5pt for cosine formula]

$\therefore$ Distance to the human habitat is 2.4525 nautical miles [0.5 pt for answer]

Solution e

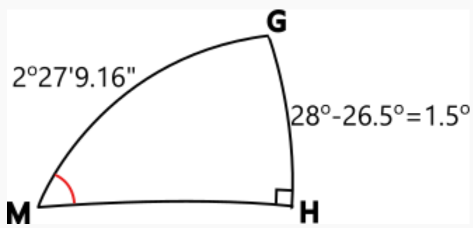


Figure 11: Sine rule

$$\frac{\sin \angle GMH}{\sin GH} = \frac{\sin 90^\circ}{\sin MG}$$

[0.5 pt for sine rule]

$$\angle GMH \approx 37.71^\circ$$

Hence, the angle should be 37.71° Northwards from East. [0.5 pt for answer]